Contents

An Introduction to Graphing Utilities

Graphing utilities such as graphing calculators and computers with graphing software are very valuable tools for visualizing mathematical principles, verifying solutions to equations, exploring mathematical ideas, and developing mathematical models. Although graphing utilities are extremely helpful in learning mathematics, their use does not mean that learning algebra is any less important. In fact, the combination of knowledge of mathematics and the use of graphing utilities allows you to explore mathematics more easily and to a greater depth. If you are using a graphing utility in this course, it is up to you to learn its capabilities and to practice using this tool to enhance your mathematical learning.

In this text there are many opportunities to use a graphing utility, some of which are described below.

Some Uses of a Graphing Utility

A graphing utility can be used to

- check or validate answers to problems obtained using algebraic methods.
- discover and explore algebraic properties, rules, and concepts.
- graph functions, and approximate solutions to equations involving functions.
- efficiently perform complicated mathematical procedures such as those found in many real-life applications.
- find mathematical models for sets of data.

In this introduction, the features of graphing utilities are discussed from a generic perspective. To learn how to use the features of a specific graphing utility, consult your user's manual or the website for this text found at *college.hmco.com*. Additionally, keystroke guides are available for most graphing utilities, and your college library may have a videotape on how to use your graphing utility.

The Equation Editor

Many graphing utilities are designed to act as "function graphers." In this course, you will study functions and their graphs in detail. You may recall from previous courses that a function can be thought of as a rule that describes the relationship between two variables. These rules are frequently written in terms of x and y. For example, the equation $y = 3x + 5$ represents y as a function of x.

Many graphing utilities have an equation editor that requires an equation to be written in "$y =$" form in order to be entered, as shown in Figure 1. (You should note that your equation editor screen may not look like the screen shown in Figure 1.) To determine exactly how to enter an equation into your graphing utility, consult your user's manual.

```
 Plot1 Plot2 Plot3
\Y1=3x+5
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Figure 1

The Table Feature

Most graphing utilities are capable of displaying a table of values with x-values and one or more corresponding y-values. These tables can be used to check solutions of an equation and to generate ordered pairs to assist in graphing an equation.

To use the *table* feature, enter an equation into the equation editor in "$y =$" form. The table may have a setup screen, which allows you to select the starting x-value and the table step or x-increment. You may then have the option of automatically generating values for x and y or building your own table using the *ask* mode. In the *ask* mode, you enter a value for x and the graphing utility displays the y-value.

For example, enter the equation

$$y = \frac{3x}{x + 2}$$

into the equation editor, as shown in Figure 2. In the table setup screen, set the table to start at $x = -4$ and set the table step to 1. When you view the table, notice that the first x-value is -4 and each value after it increases by 1. Also notice that the Y_1 column gives the resulting y-value for each x-value, as shown in Figure 3. The table shows that the y-value when $x = -2$ is ERROR. This means that the variable x may not take on the value -2 in this equation.

```
Plot1 Plot2 Plot3
\Y1=3x/(X+2)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Figure 2

X	Y1	
-4	6	
-3	9	
-2	ERROR	
-1	-3	
0	0	
1	1	
2	1.5	
X=-4		

Figure 3

With the same equation in the equation editor, set the table to *ask* mode. In this mode you do not need to set the starting x-value or the table step, because you are entering any value you choose for x. You may enter any real value for x—integers, fractions, decimals, irrational numbers, and so forth. If you enter $x = 1 + \sqrt{3}$, the graphing utility may rewrite the number as a decimal approximation, as shown in Figure 4. You can continue to build your own table by entering additional x-values in order to generate y-values.

X	Y1	
2.7321	1.7321	
X=		

Figure 4

If you have several equations in the equation editor, the table may generate y-values for each equation.

Creating a Viewing Window

A **viewing window** for a graph is a rectangular portion of the coordinate plane. A viewing window is determined by the following six values.

Xmin = the smallest value of x

Xmax = the largest value of x

Xscl = the number of units per tick mark on the x-axis

Ymin = the smallest value of y

Ymax = the largest value of y

Yscl = the number of units per tick mark on the y-axis

When you enter these six values into a graphing utility, you are setting the viewing window. Some graphing utilities have a standard viewing window, as shown in Figure 5.

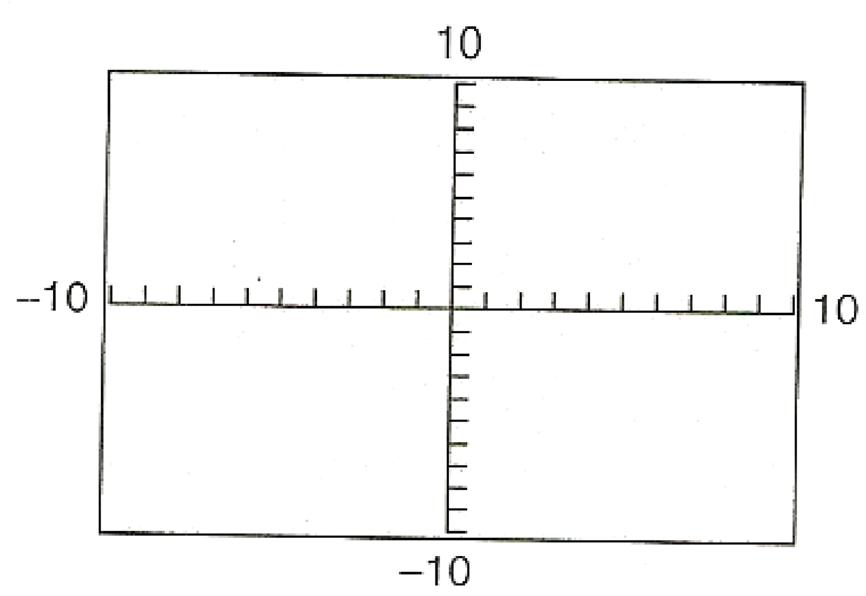
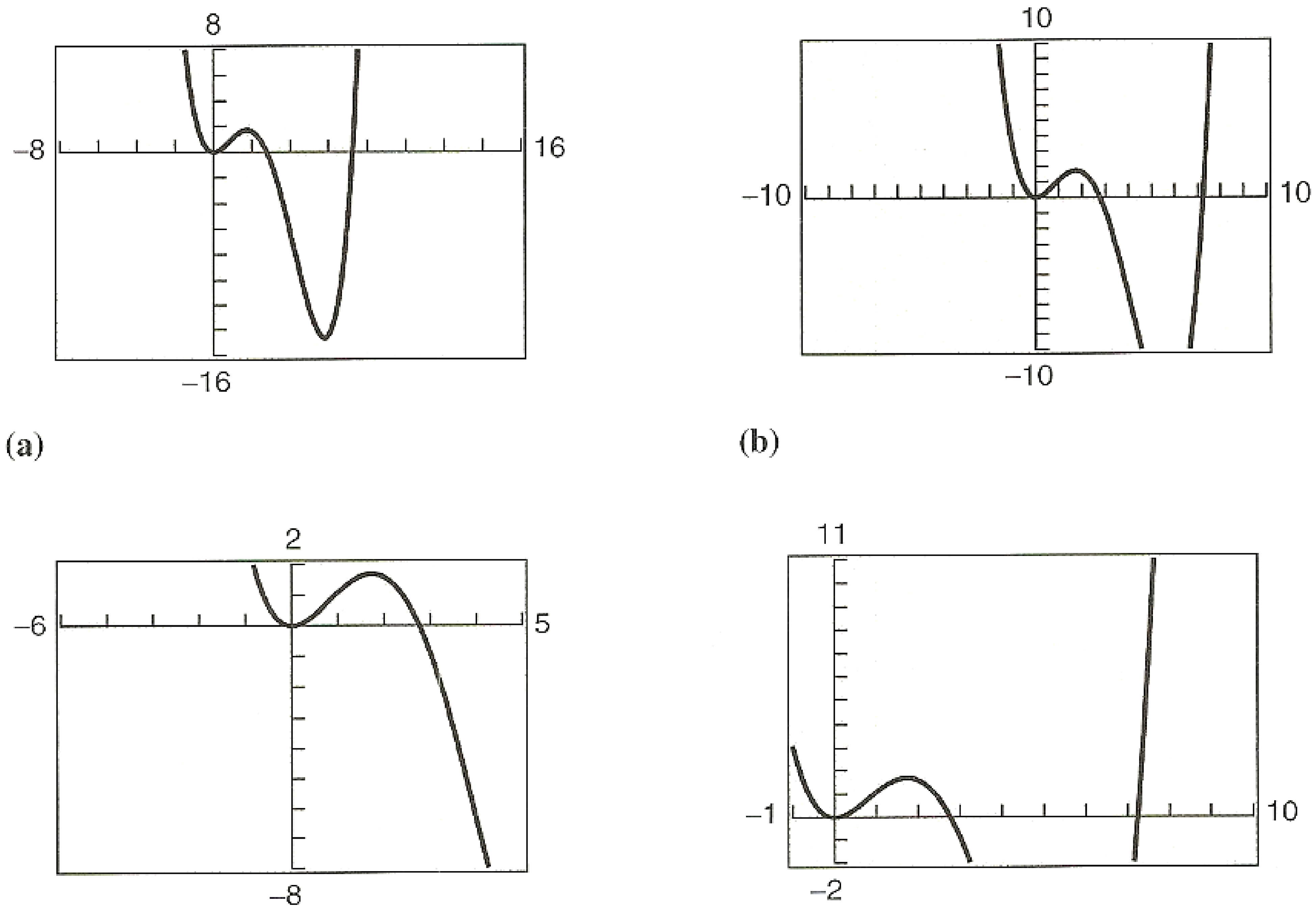


Figure 5

By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure 6 shows four different viewing windows for the graph of

$$y = 0.1x^4 - x^3 + 2x^2.$$

Of these, the view shown in part (a) is the most complete.



(a) (b) (c) (d)

Figure 6

On most graphing utilities, the display screen is two-thirds as high as it is wide. On such screens, you can obtain a graph with a true geometric perspective by using **a square setting**—one in which

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{2}{3}.$$

One such setting is shown in Figure 7. Notice that the x and y tick marks are equally spaced on a square setting, but not on a standard setting.

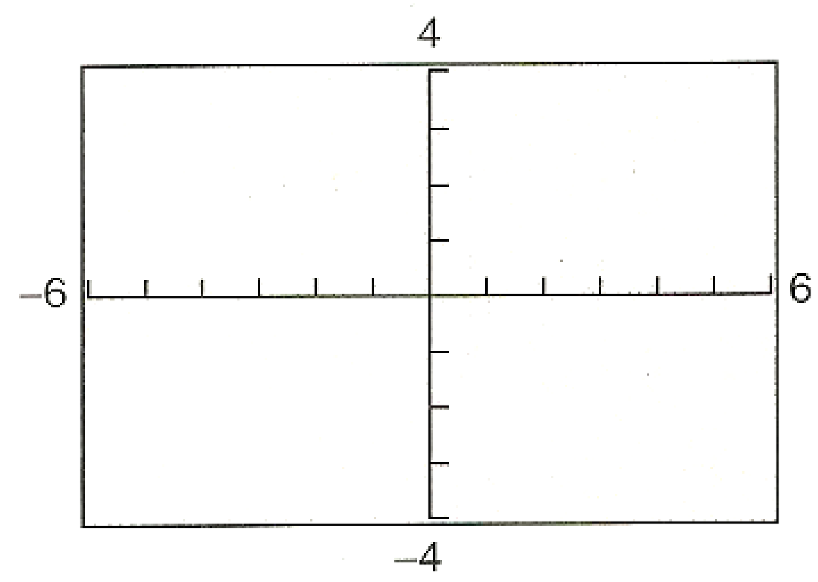


Figure 7

To see how the viewing window affects the geometric perspective, graph the semicircles $y_1 = \sqrt{9 - x^2}$ and $y_2 = -\sqrt{9 - x^2}$ in a standard viewing window. Then graph y_1 and y_2 in a square window. Note the difference in the shapes of the circles.

Zoom and Trace Features

When you graph an equation, you can move from point to point along its graph using the *trace* feature. As you trace the graph, the coordinates of each point are displayed, as shown in Figure 8. The *trace* feature combined with the *zoom* feature allows you to obtain better and better approximations of desired points on a graph. For instance, you can use the *zoom* feature of a graphing utility to approximate the x-intercept(s) of a graph [the point(s) where the graph crosses the x-axis]. Suppose you want to approximate the x-intercept(s) of the graph of $y = 2x^3 - 3x + 2$.

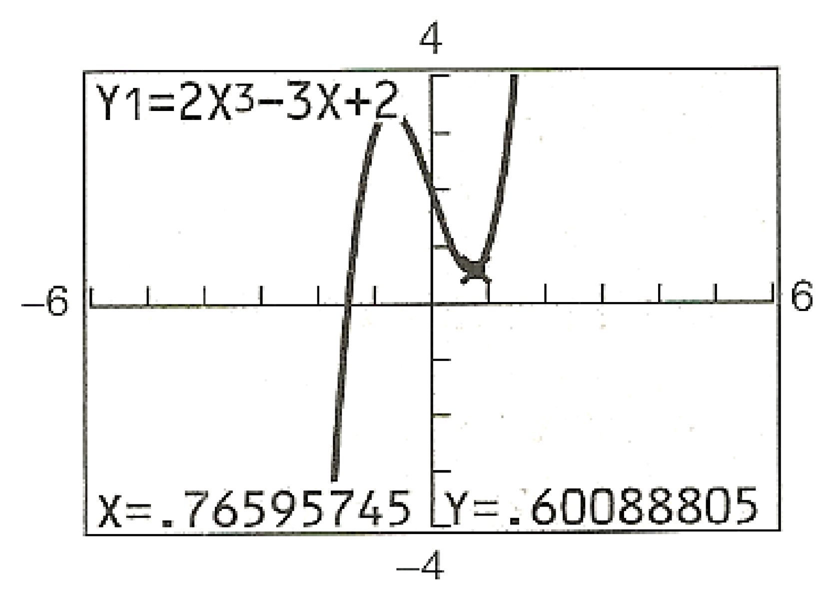
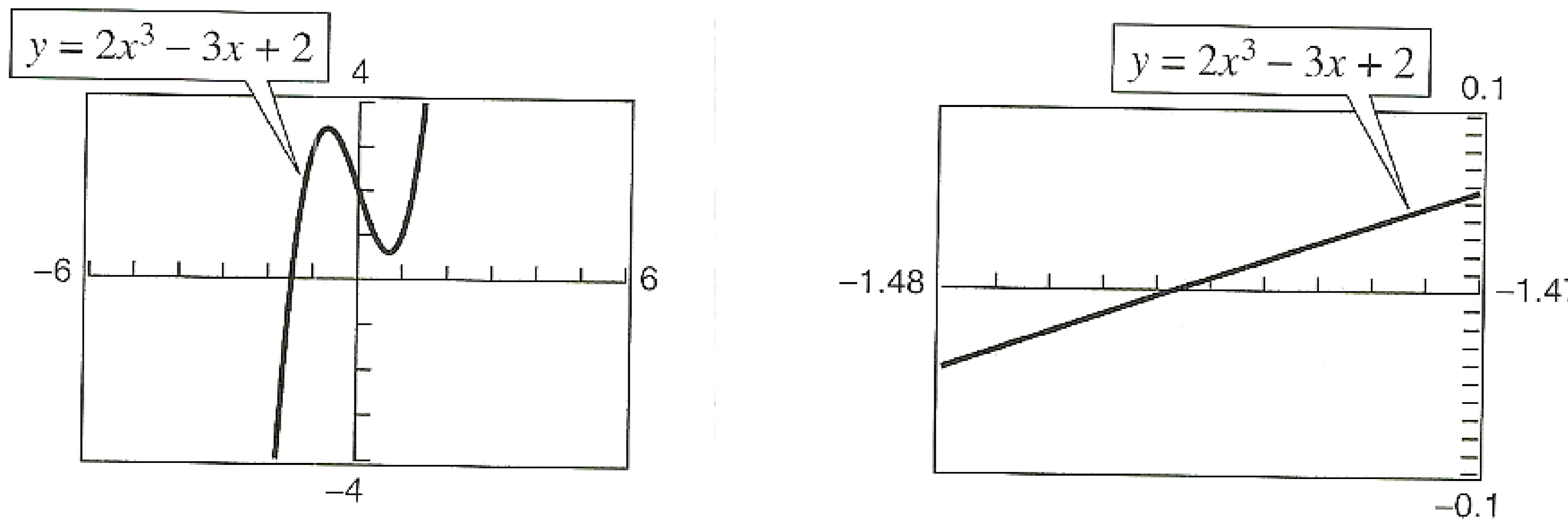


Figure 8

Begin by graphing the equation, as shown in Figure 9(a). From the viewing window shown, the graph appears to have only one x-intercept. This intercept lies between -2 and -1. By zooming in on the intercept, you can improve the approximation, as shown in Figure 9(b). To three decimal places, the solution is $x \approx -1.476$.



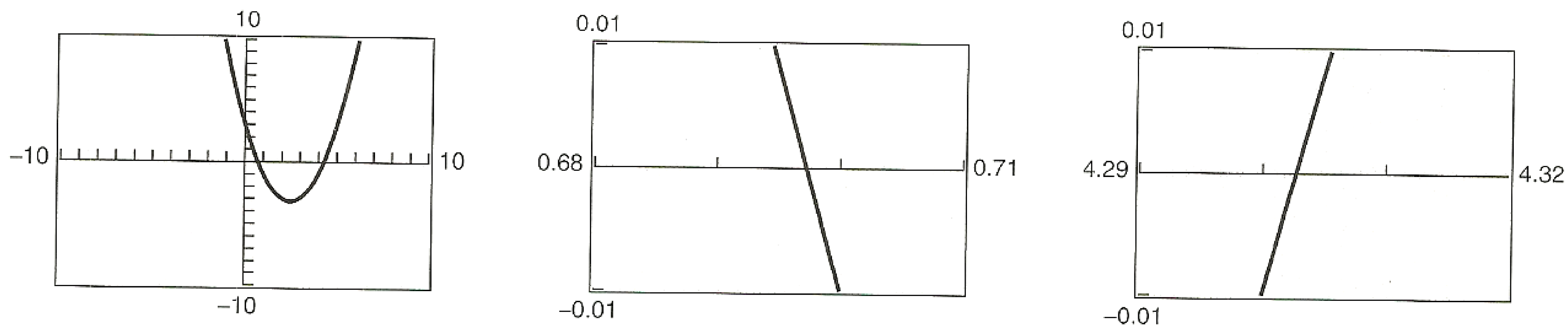
(a) (b)

Figure 9

Here are some suggestions for using the *zoom* feature.

1. With each successive zoom-in, adjust the x-scale so that the viewing window shows at least one tick mark on each side of the x-intercept.
2. The error in your approximation will be less than the distance between two scale marks.
3. The *trace* feature can usually be used to add one more decimal place of accuracy without changing the viewing window.

Figure 10(a) shows the graph of $y = x^2 - 5x + 3$. Figures 10(b) and 10(c) show "zoom-in views" of the two x-intercepts. From these views, you can approximate the x-intercepts to be $x \approx 0.697$ and $x \approx 4.303$.

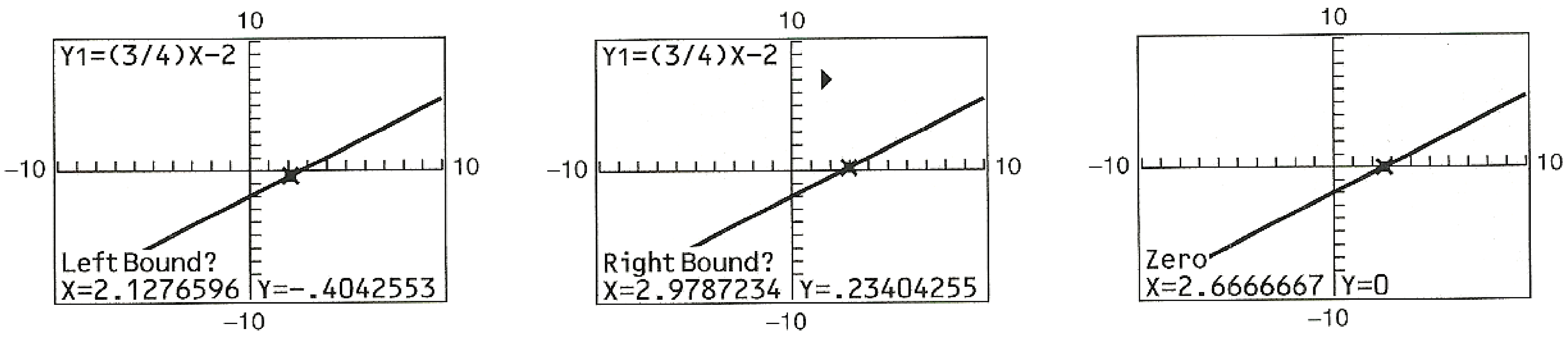


(a) (b) (c)

Figure 10

Zero or Root Feature

Using the *zero* or *root* feature, you can find the real zeros of functions of the various types studied in this text—polynomial, exponential, logarithmic, and trigonometric functions. To find the zeros of a function such as $f(x) = \frac{3}{4}x - 2$, first enter the function as $y_1 = \frac{3}{4}x - 2$. Then use the *zero* or *root* feature, which may require entering lower and upper bound estimates of the root, as shown in Figure 11.



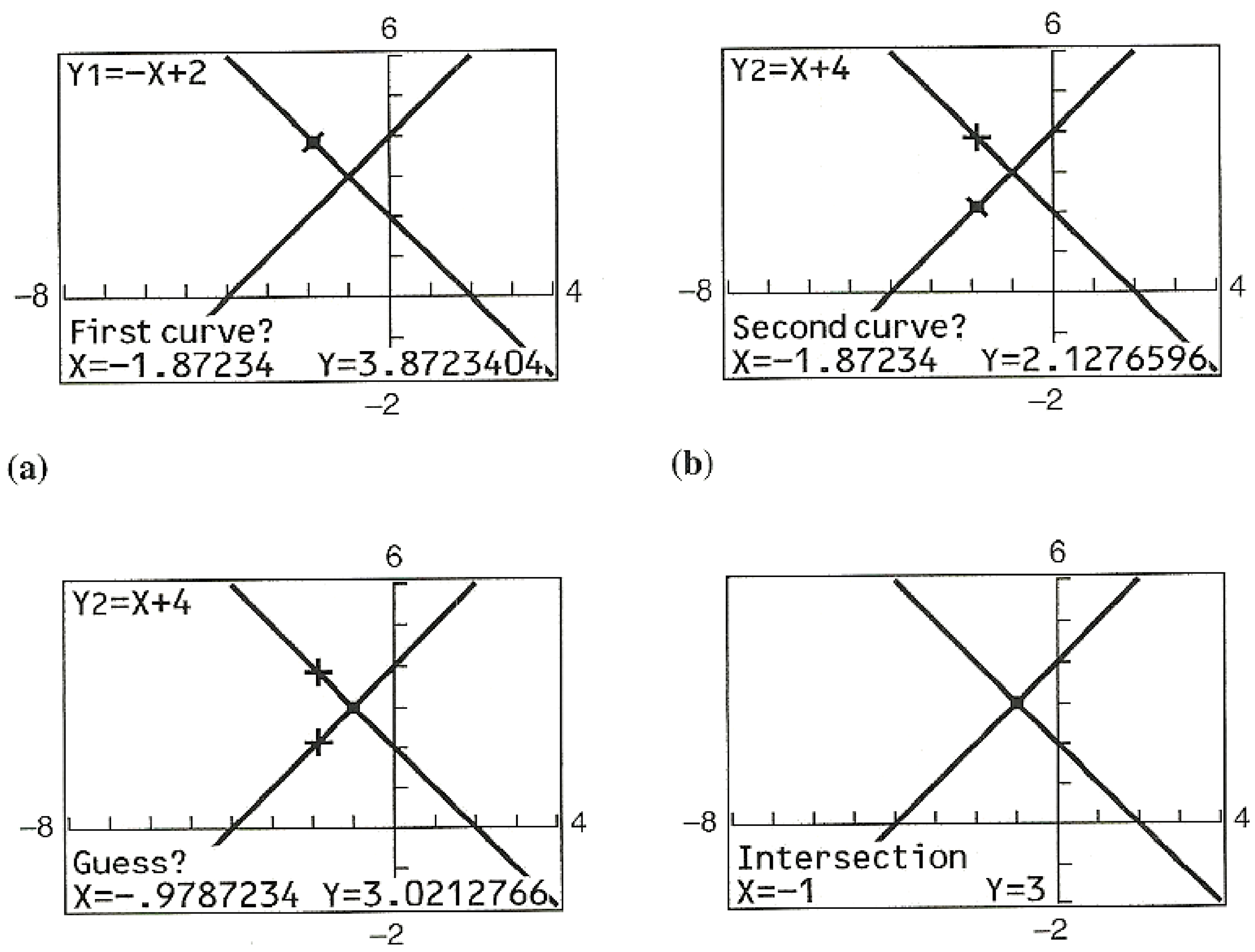
(a) (b) (c)

Figure 11

In Figure 11(c), you can see that the zero is $x = 2.6666667 \approx 2\frac{2}{3}$.

Intersect Feature

To find the points of intersection of two graphs, you can use the *intersect* feature. For instance, to find the points of intersection of the graphs of $y_1 = -x + 2$ and $y_2 = x + 4$, enter these two functions and use the *intersect* feature, as shown in Figure 12.



(a) (b)

(c) (d)

Figure 12

From Figure 12(d), you can see that the point of intersection is $(-1, 3)$.

Regression Capabilities

Throughout the text, you will be asked to use the regression capabilities of a graphing utility to find models for sets of data. Most graphing utilities have built-in regression programs for the following.

Regression	*Form of Model*
Linear	$y = ax + b$
Quadratic	$y = ax^2 + bx + c$
Cubic	$y = ax^3 + bx^2 + cx + d$
Quartic	$y = ax^4 + bx^3 + cx^2 + dx + e$
Logarithmic	$y = a + b\ln(x)$
Exponential	$y = ab^x$
Power	$y = ax^b$
Logistic	$y = \dfrac{c}{1 + ae^{-bx}}$
Sine	$y = a\sin(bx + c) + d$

For instance, you can find the linear regression model for the average hourly wages y (in dollars per hour) of production workers in manufacturing industries from 1987 through 1997 shown in the table. (Source: U.S. Bureau of Labor Statistics)

Year	1987	1988	1989	1990	1991	1992
y	9.91	10.19	10.48	10.83	11.18	11.46

Year	1993	1994	1995	1996	1997
y	11.74	12.06	12.37	12.78	13.17

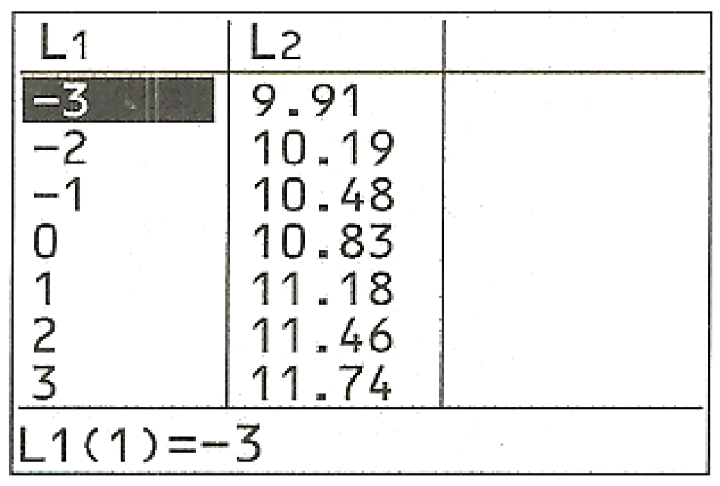


Figure 13

First, let $x = 0$ correspond to 1990 and enter the data into the list editor, as shown in Figure 13. Note that the list in the first column contains the years and the list in the second column contains the hourly wages that correspond to the years. Run your graphing utility's built-in linear regression program to obtain the coefficients a and b for the model $y = ax + b$, as shown in Figure 14. So, a linear model for the data is

$$y \approx 0.321x + 10.83.$$

When you run some regression programs, you may obtain an "r-value," which gives a measure of how well the model fits the data. The closer the value of $|r|$ is to 1, the better the fit. For the data in the table above, $r \approx 0.999$, which implies that the model is a very good fit.

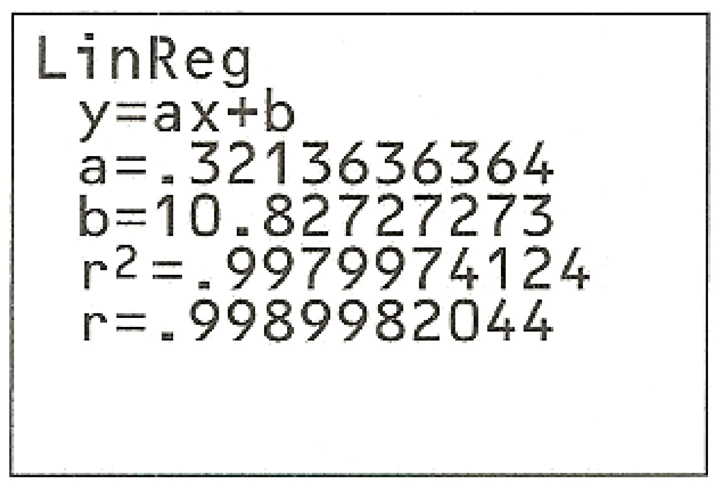
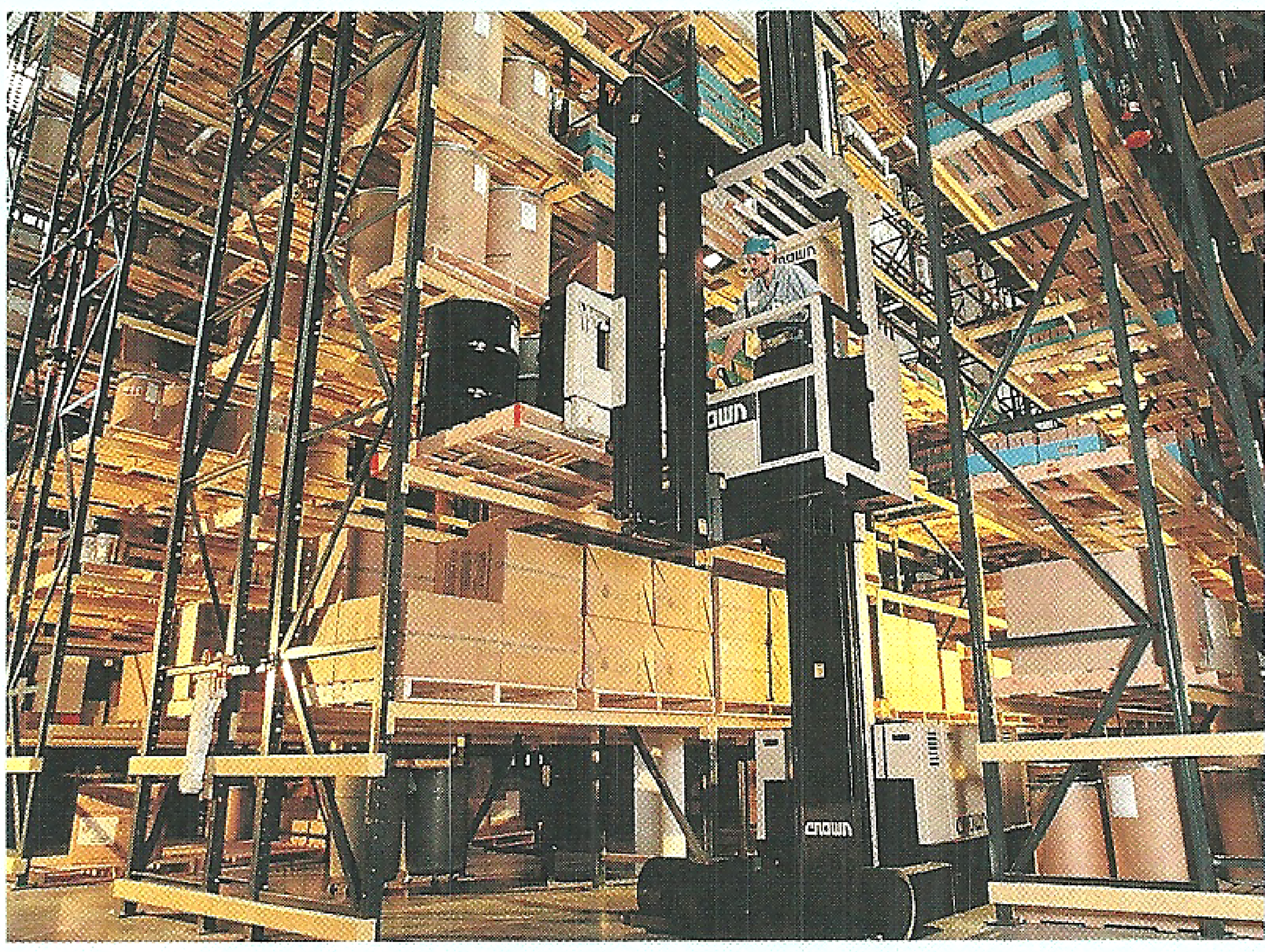


Figure 14

Prerequisites



In 1997, over $24 billion worth of corrugated and solid fiber boxes were produced to create shipping containers. (Source: U.S. Bureau of the Census)

The Big Picture

In this chapter you will learn how to

- plot points in the coordinate plane and use the Distance and Midpoint Formulas.
- sketch graphs of equations by point-plotting or using a graphing utility.
- find and use the slope of a line to write and graph linear equations.
- solve linear equations, quadratic equations, polynomial equations, equations involving radicals, equations involving fractions, and equations involving absolute values.
- solve linear inequalities, inequalities involving absolute values, polynomial inequalities, and rational inequalities.

Important Vocabulary

As you encounter each new vocabulary term in this chapter, add the term and its definition to your notebook glossary.

- rectangular coordinate system (p. 2)
- Cartesian plane (p. 2)
- Distance Formula (p. 5)
- Midpoint Formula (p. 7)
- circle of radius r (p. 8)
- standard form of the equation of a circle (p. 8)
- solution point (p. 14)
- graph of an equation (p. 14)
- intercepts (p. 15)
- slope (p. 25)
- point-slope form (p. 27)
- slope-intercept form (p. 29)
- general form (p. 30)
- parallel (p. 31)
- perpendicular (p. 31)
- equation (p. 38)
- solution (p. 38)
- linear equation in one variable x (p. 38)
- extraneous (p. 39)
- x-intercept (p. 39)
- y-intercept (p. 39)
- point of intersection (p. 43)
- solutions of an inequality (p. 54)
- graph of an inequality (p. 54)
- properties of inequalities (p. 54)
- equivalent inequalities (p. 54)
- linear inequality (p. 55)
- double inequality (p. 56)
- critical numbers (p. 58)
- test intervals (p. 58)

Additional Resources Text-specific additional resources are available to help you do well in this course. See page xvi for details.

P.1 Graphical Representation of Data

The Cartesian Plane

Just as you can represent real numbers by points on a real number line, you can represent ordered pairs of real numbers by points in a plane called the **rectangular coordinate system,** or the **Cartesian plane,** after the French mathematician René Descartes (1596–1650).

The Cartesian plane is formed by using two real number lines intersecting at right angles, as shown in Figure P.1. The horizontal real number line is usually called the ***x*-axis,** and the vertical real number line is usually called the ***y*-axis.** The point of intersection of these two axes is the **origin,** and the two axes divide the plane into four parts called **quadrants.**

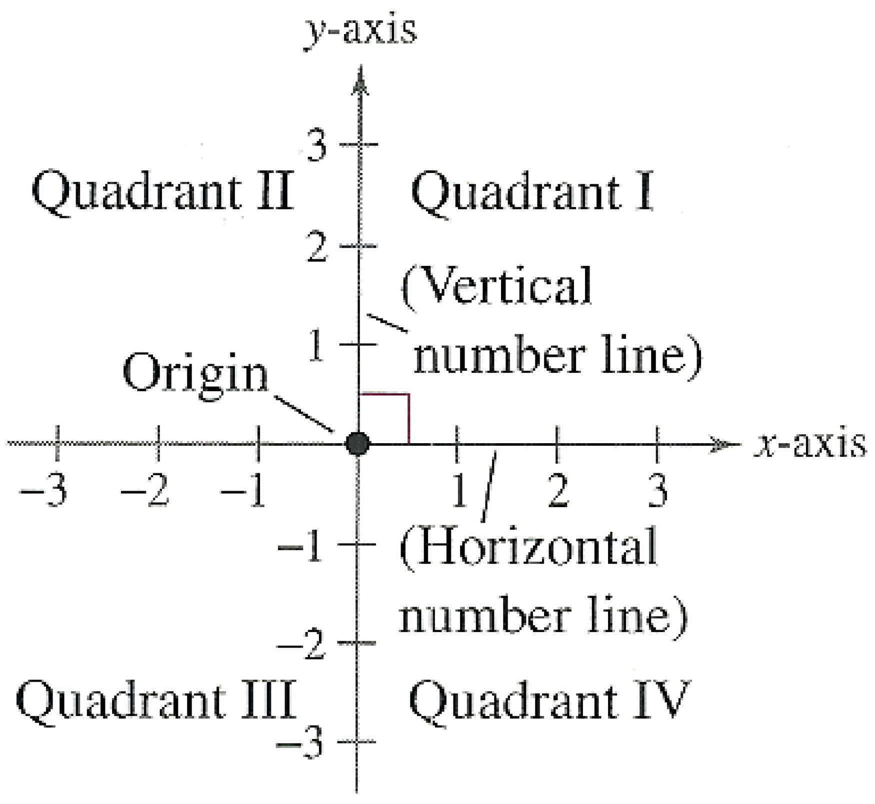


Figure P.1 *The Cartesian Plane*

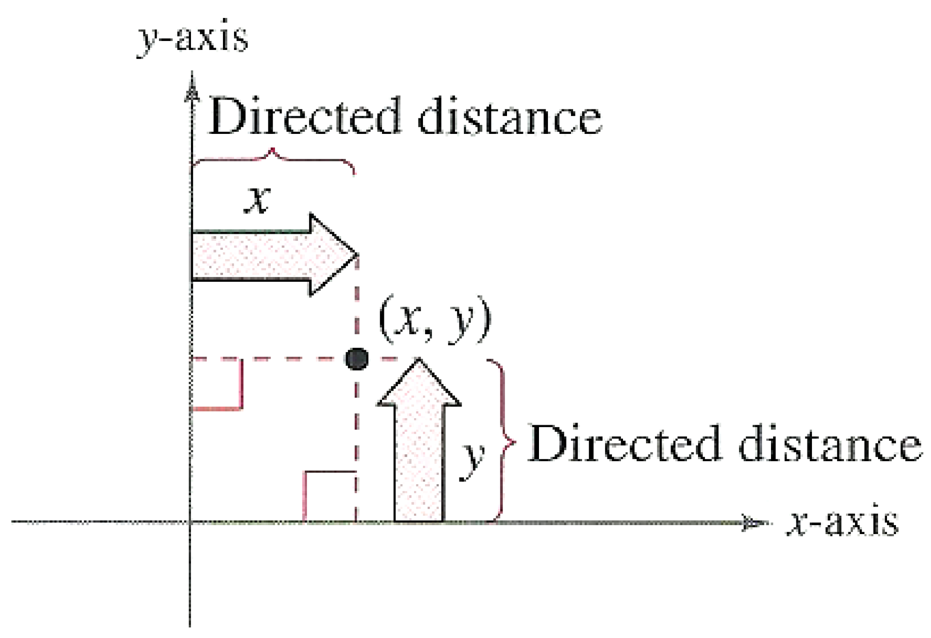


Figure P.2 *Ordered Pair (x, y)*

Each point in the plane corresponds to an **ordered pair** (x, y) of real numbers x and y, called **coordinates** of the point. The ***x*-coordinate** represents the directed distance from the y-axis to the point, and the ***y*-coordinate** represents the directed distance from the x-axis to the point, as shown in Figure P.2.

Directed distance from y-axis → (x, y) ← Directed distance from x-axis

The notation (x, y) denotes both a point in the plane and an open interval on the real number line. The context will tell you which meaning is intended.

What You Should Learn:

- How to plot points in the Cartesian plane
- How to represent data graphically using scatter plots, bar graphs, and line graphs
- How to use the Distance Formula to find the distance between two points
- How to use the Midpoint Formula to find the midpoint of a line segment
- How to find the equation of a circle

Why You Should Learn It:

The Cartesian plane can be used to represent relationships between two variables. For instance, Exercise 87 on page 13 shows how to graphically represent the number of recording artists elected to the Rock and Roll Hall of Fame from 1986 to 1999.

EXAMPLE 1 Plotting Points in the Cartesian Plane

Plot the points $(-1, 2)$, $(3, 4)$, $(0, 0)$, $(3, 0)$, and $(-2, -3)$.

Solution

To plot the point $(-1, 2)$, imagine a vertical line through -1 on the x-axis and a horizontal line through 2 on the y-axis. The intersection of these two lines is the point $(-1, 2)$. This point is 1 unit to the left of the y-axis and 2 units up from the x-axis. The other four points can be plotted in a similar way (see Figure P.3).

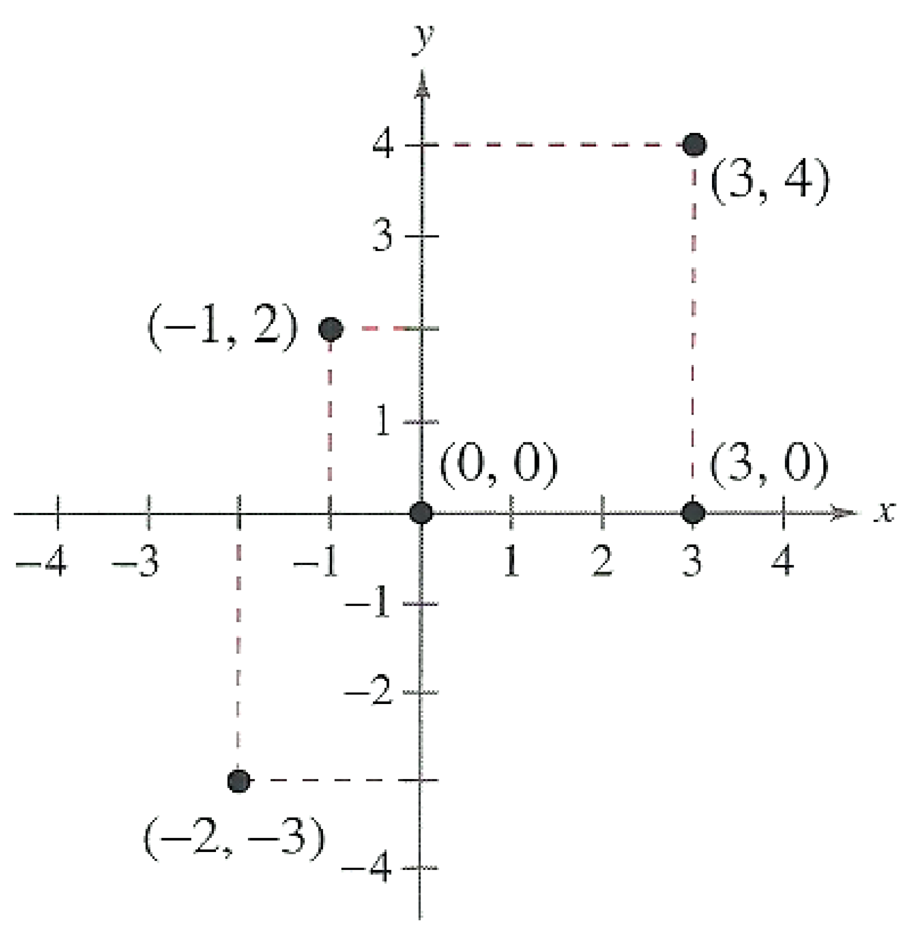
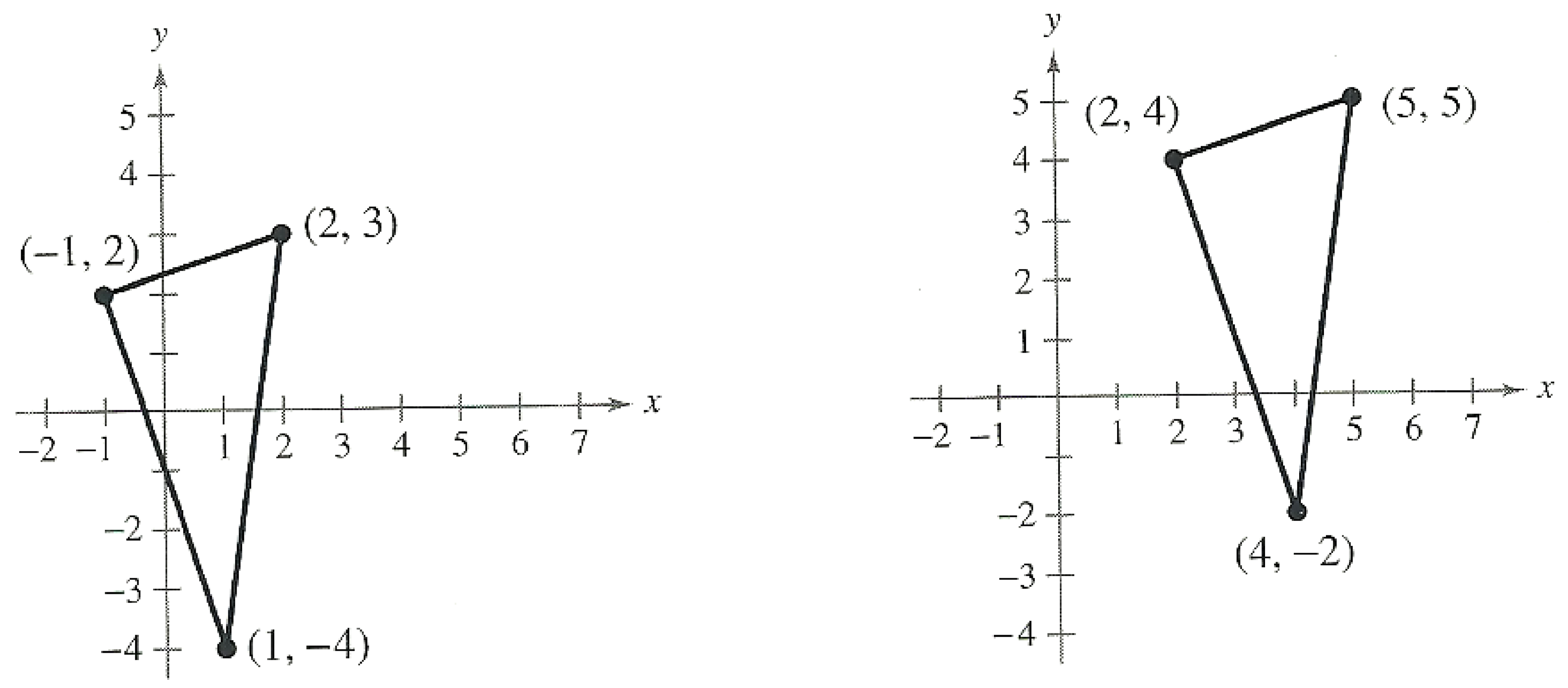


Figure P.3

EXAMPLE 2 Translating Points in the Plane

The triangle in Figure P.4(a) has vertices at the points $(-1, 2)$, $(1, -4)$, and $(2, 3)$. Shift the triangle three units to the right and two units up and find the vertices of the shifted triangle, as shown in Figure P.4(b).



(a) (b)

Figure P.4

Solution

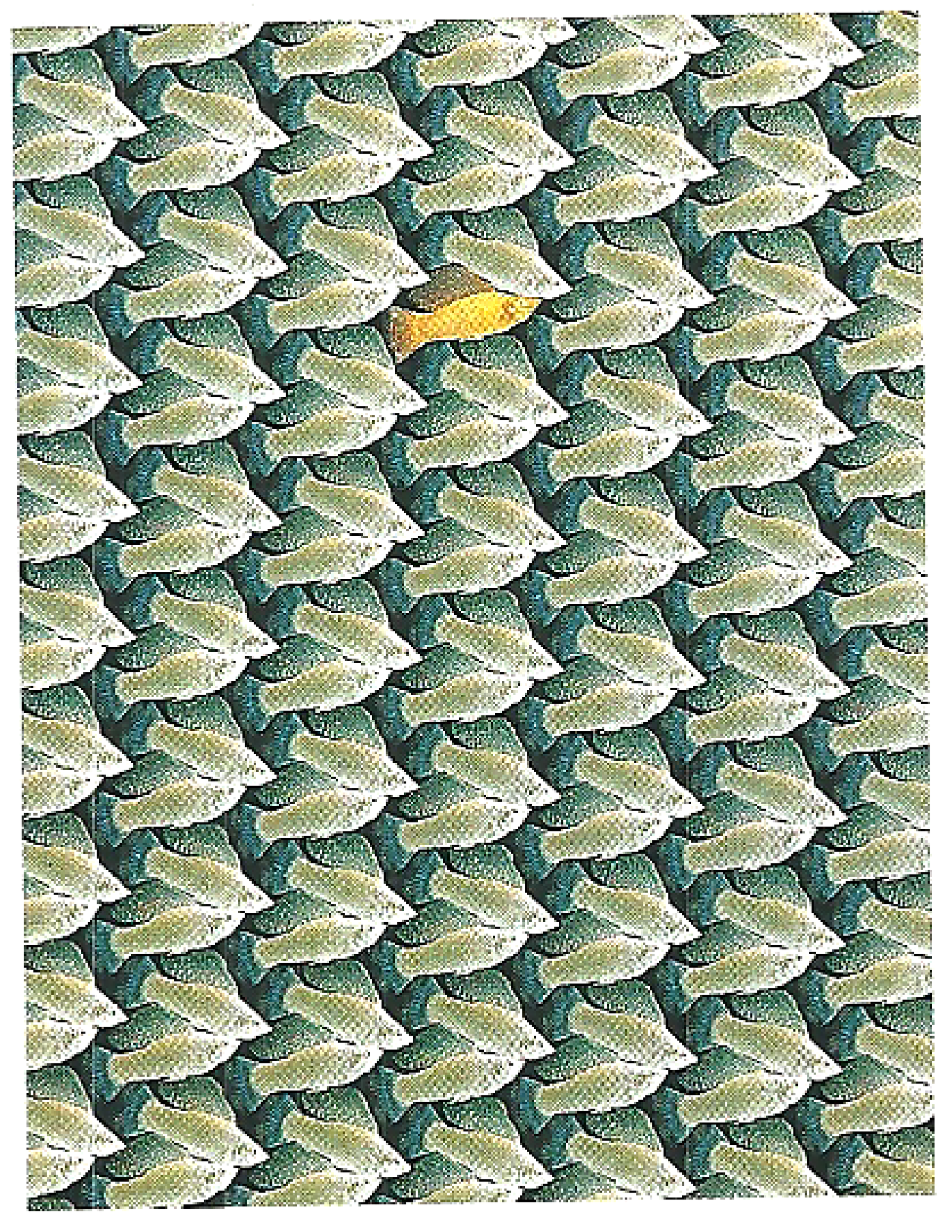
To shift the vertices three units to the right, add 3 to each of the x-coordinates. To shift the vertices two units up, add 2 to each of the y-coordinates.

Original Point	*Translated Point*
$(-1, 2)$	$(-1 + 3, 2 + 2) = (2, 4)$
$(1, -4)$	$(1 + 3, -4 + 2) = (4, -2)$
$(2, 3)$	$(2 + 3, 3 + 2) = (5, 5)$

Plotting the translated points and sketching the line segments between them produces the shifted triangle shown in Figure P.4(b).

Example 2 shows how to translate points in a coordinate plane. The following transformed points are related to the original points as follows.

Original Point	*Transformed Point*	
(x, y)	$(-x, y)$	$(-x, y)$ is a reflection of the original point in the y-axis.
(x, y)	$(x, -y)$	$(x, -y)$ is a reflection of the original point in the x-axis.
(x, y)	$(-x, -y)$	$(-x, -y)$ is a reflection of the original point through the origin.



Paul Morrell

Much of computer graphics, including this computer-generated goldfish tessellation, consists of transformations of points in a coordinate plane. One type of transformation, a translation, is illustrated in Example 2. Other types include reflections, rotations, and stretches.

A computer animation of this example appears in the *Interactive* CD-ROM and *Internet* versions of this text.

Representing Data Graphically

The beauty of a rectangular coordinate system is that it allows you to see relationships between two variables. It would be difficult to overestimate the importance of Descartes's introduction of coordinates to the plane. Today, his ideas are in common use in virtually every scientific and business-related field.

EXAMPLE 3 Sketching a Scatter Plot, Bar Graph, and Line Graph

From 1988 through 1997, the amount A (in millions of dollars) spent on archery equipment in the United States is given in the table, where t represents the year. (a) Sketch a scatter plot of the data. (b) Sketch a bar graph of the data. (c) Sketch a line graph of the data. (Source: National Sporting Goods Association)

t	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997
A	235	261	265	270	334	285	306	287	272	273

A computer animation of this example appears in the *Interactive* CD-ROM and *Internet* versions of this text.

Solution

a. To sketch a *scatter plot* of the data given in the table, you simply represent each pair of values by an ordered pair (t, A) and plot the resulting points, as shown in Figure P.5. For instance, the first pair of values is represented by the ordered pair (1988, 235). Note that the break in the t-axis indicates that the numbers between 0 and 1988 have been omitted.

b. To create a *bar graph*, begin by drawing a vertical axis to represent the amount (in millions of dollars) and a horizontal axis to represent the year. Then for each value of t in the table, draw a vertical bar whose height is the corresponding value A. The bar graph is shown in Figure P.6.

c. To draw a *line graph*, begin by drawing a vertical axis to represent the amount (in millions of dollars). Then label the horizontal axis with years and plot the points given in the table. Finally, connect the points with line segments, as shown in Figure P.7.

STUDY TIP

You can use a graphing utility to sketch each of the graphs in Example 3. First, enter the data into the graphing utility. You can then use the *statistical plotting* feature to sketch a scatter plot, a bar graph, and a line graph.

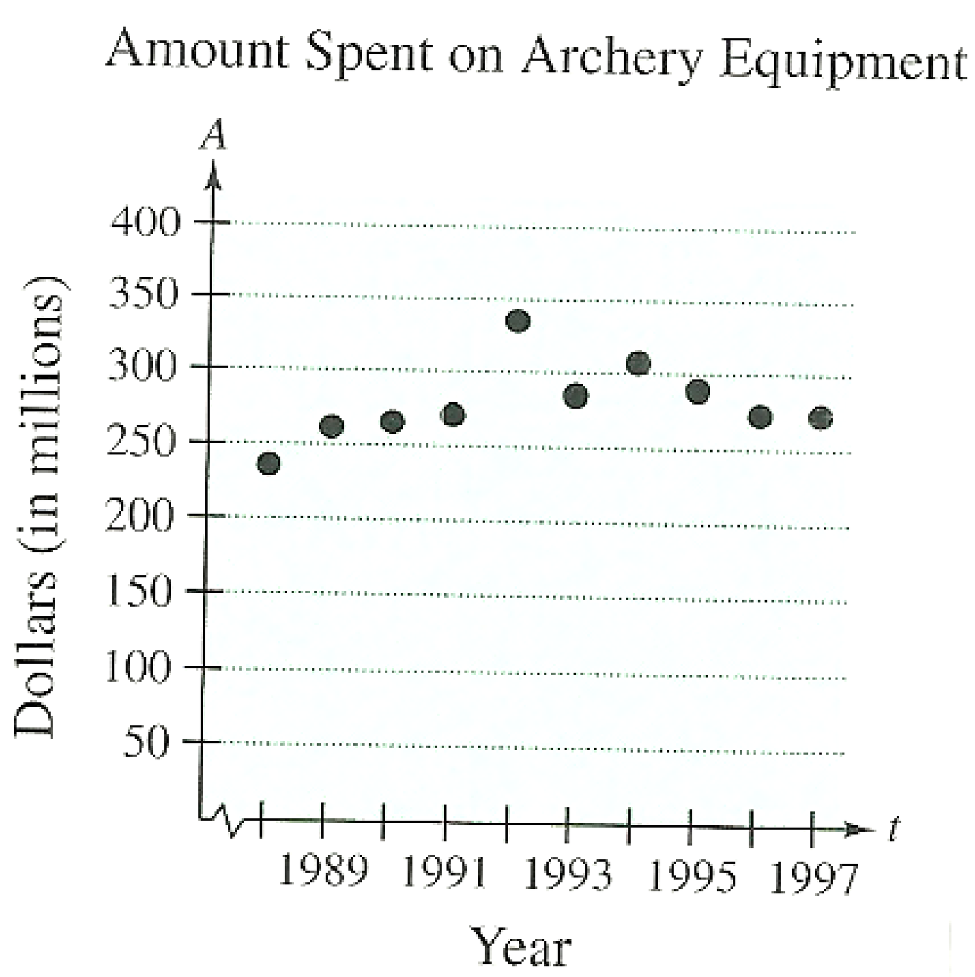


Figure P.5

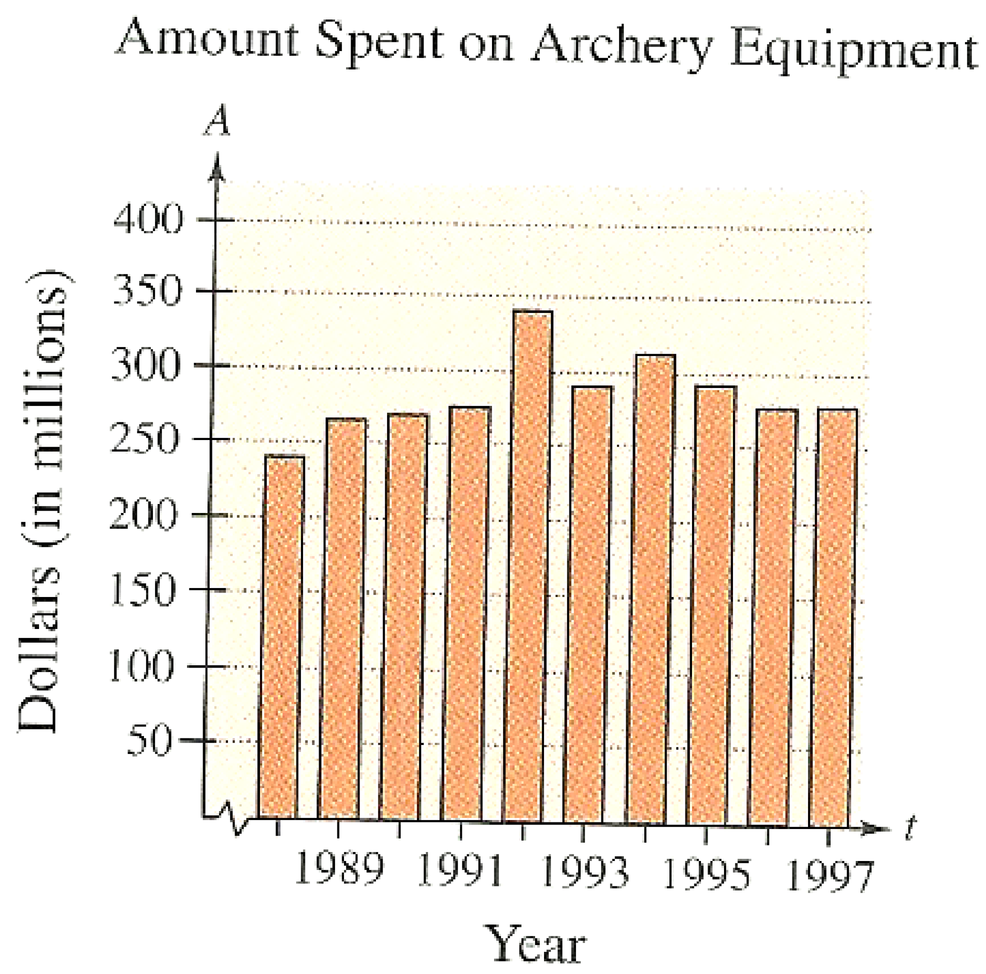


Figure P.6

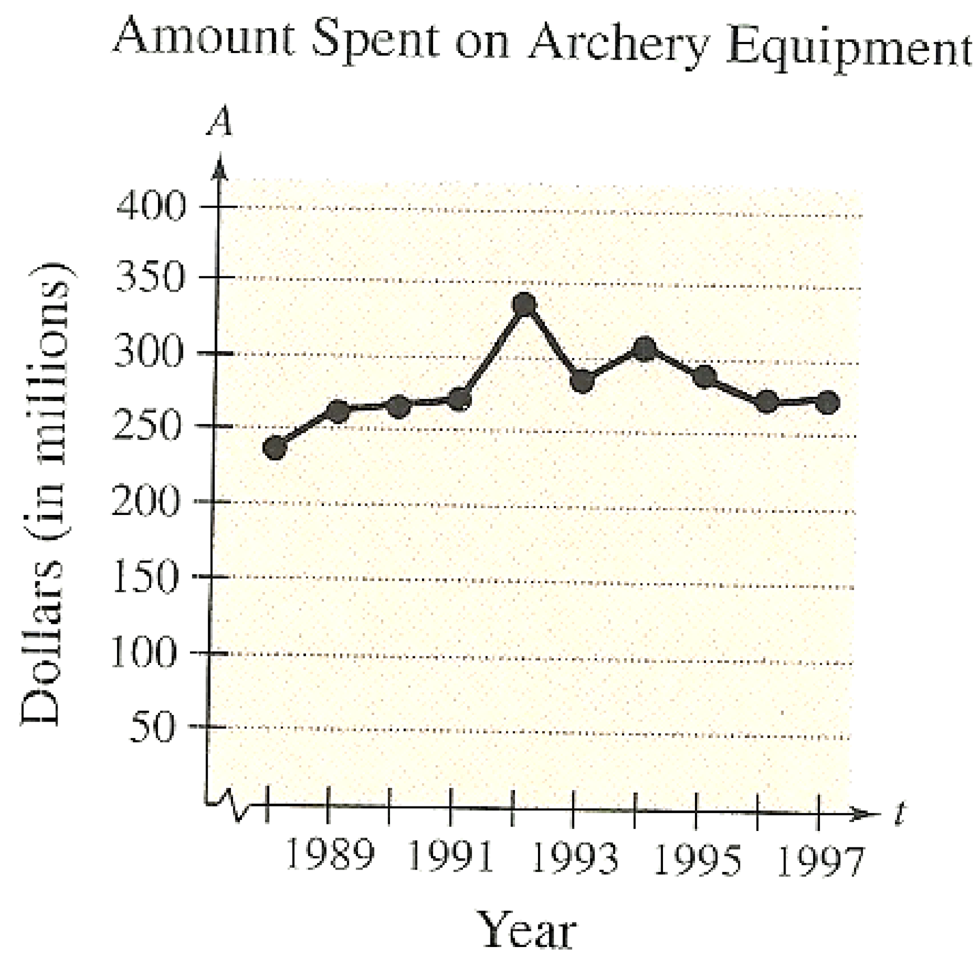


Figure P.7

In Example 3, you could have let $t = 1$ represent the year 1988. In that case, the horizontal axis of each graph would not have been broken, and the tick marks would have been labeled 1 through 10 (instead of 1988 through 1997).

EXAMPLE 4 Interpreting a Population Model

The population (in millions) of North Carolina from 1990 to 1997 can be modeled by $y = 0.11x + 6.63$, where x is the time in years, with $x = 0$ corresponding to 1990. During which year did the population exceed 7 million?

Graphical Solution

You can use the *statistical plotting* feature of a graphing utility to create a bar graph of the population values obtained using the model. On the same viewing window, graph the line $y = 7$.

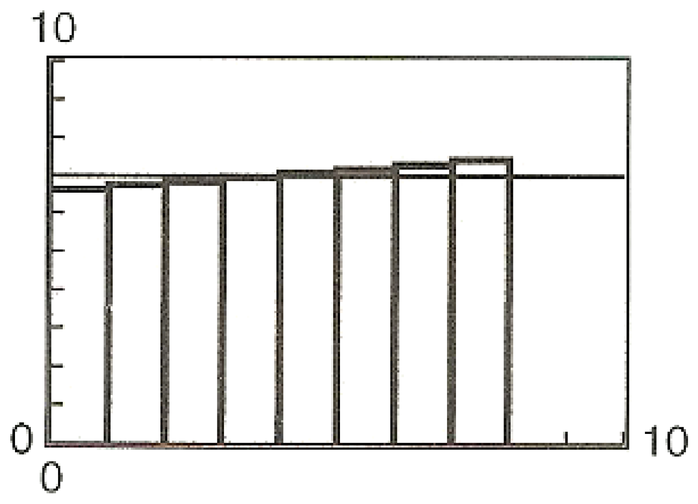


Figure P.8

From the graph in Figure P.8 you can see that the bars that correspond to $x = 0, 1, 2,$ and 3 are less than 7. The bar that corresponds to $x = 4$ is the first bar greater than 7, so the population exceeded 7 million sometime during 1993.

Numerical Solution

You can use the *table* feature of a graphing utility to evaluate the model $y = 0.11x + 6.63$ for different years. Enter the model into the equation editor. Then create a table such as the one below.

X	Y1	
0	6.63	
1	6.74	
2	6.85	
3	6.96	
4	7.07	
5	7.18	
6	7.29	
X=3		

Figure P.9

From the table in Figure P.9, you can see that the value of y when $x = 3$ is 6.96, and the value of y when $x = 4$ is 7.07. So, the population exceeded 7 million when x is greater than 3 and less than 4, or sometime during 1993.

The Distance Formula

Recall from the Pythagorean Theorem that for a right triangle with hypotenuse of length c and sides of lengths a and b, you have $a^2 + b^2 = c^2$ as shown in Figure P.10. (The converse is also true. That is, if $a^2 + b^2 = c^2$, the triangle is a right triangle.)

Suppose you want to determine the distance d between two points (x_1, y_1) and (x_2, y_2) in the plane. With these two points, a right triangle can be formed, as shown in Figure P.11. The length of the vertical side of the triangle is $|y_2 - y_1|$, and the length of the horizontal side is $|x_2 - x_1|$. By the Pythagorean Theorem,

$$d^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$
$$d = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$
$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

This result is the **Distance Formula.**

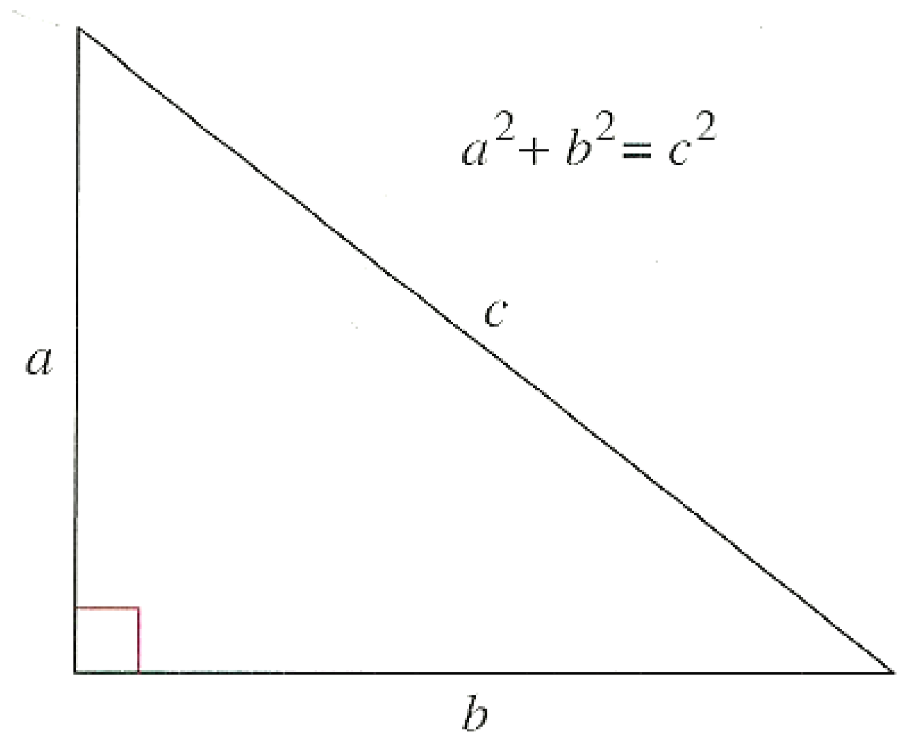


Figure P.10

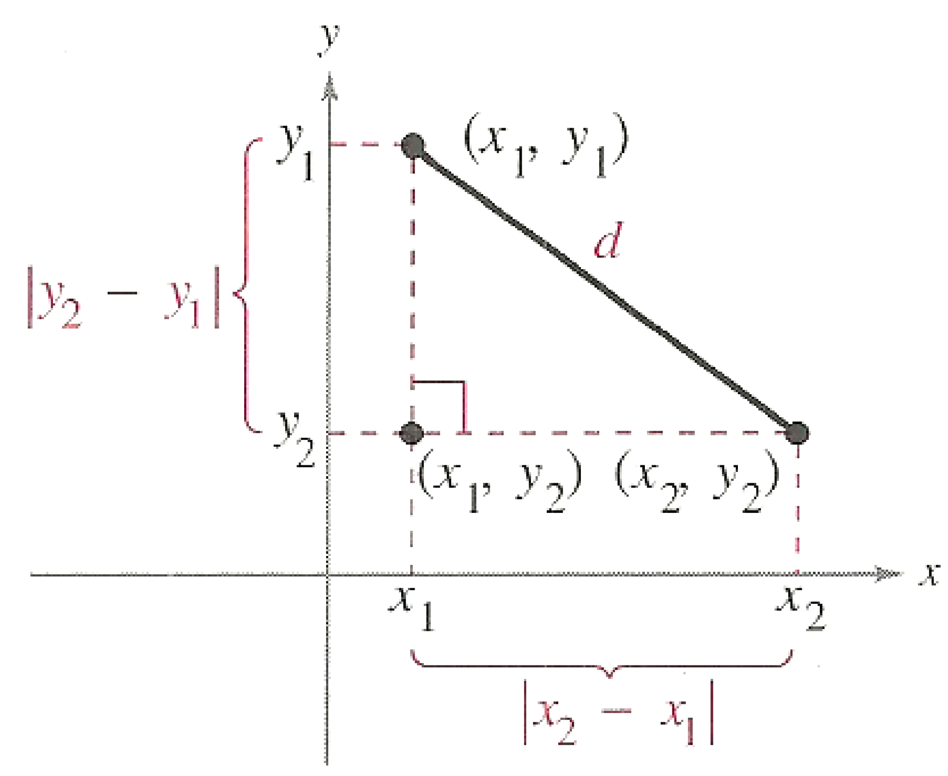


Figure P.11

The Distance Formula

The distance d between the points (x_1, y_1) and (x_2, y_2) in the plane is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

EXAMPLE 5 Finding a Distance

Find the distance between the points $(-2, 1)$ and $(3, 4)$.

Algebraic Solution

Let $(x_1, y_1) = (-2, 1)$ and $(x_2, y_2) = (3, 4)$. Then apply the Distance Formula as follows.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{Distance Formula}$$

$$= \sqrt{[3 - (-2)]^2 + (4 - 1)^2} \quad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= \sqrt{(5)^2 + (3)^2} \quad \text{Simplify.}$$

$$= \sqrt{34}$$

$$\approx 5.83 \quad \text{Use a calculator.}$$

So, the distance between the points is about 5.83 units.

You can use the Pythagorean Theorem to check that the distance is correct.

$$d^2 \stackrel{?}{=} 3^2 + 5^2 \quad \text{Pythagorean Theorem}$$

$$(\sqrt{34})^2 \stackrel{?}{=} 3^2 + 5^2 \quad \text{Substitute for } d.$$

$$34 = 34 \quad \text{Distance checks. } \checkmark$$

Graphical Solution

Use centimeter graph paper to plot the points A and B. Carefully sketch the line segment from A to B. Then use a centimeter ruler to measure the length of the segment.

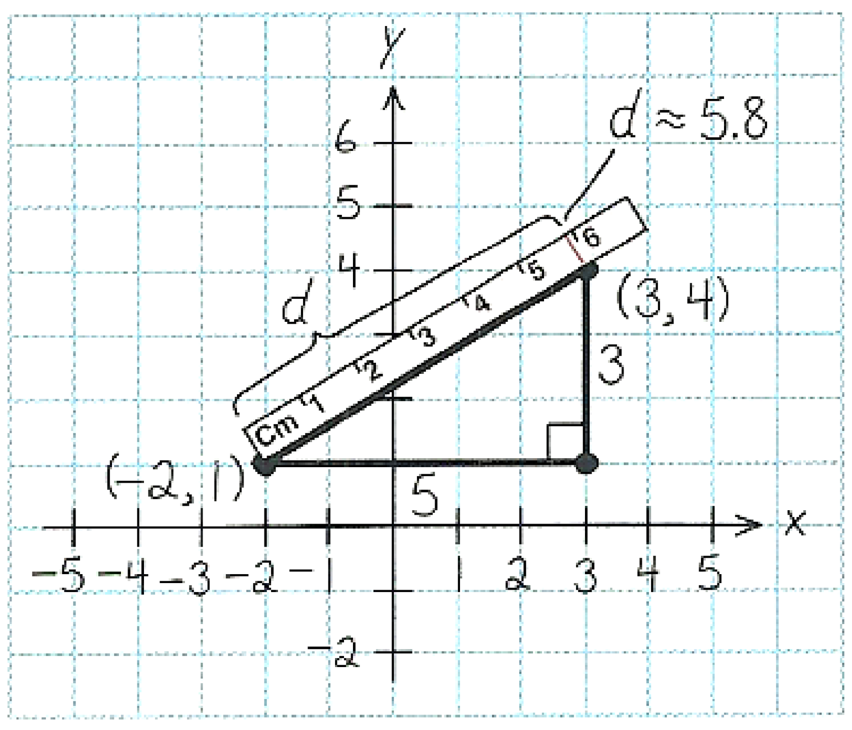


Figure P.12

The line segment measures about 5.8 centimeters as shown in Figure P.12. So, the distance between the points is about 5.8 units.

EXAMPLE 6 Verifying a Right Triangle

Show that the points $(2, 1)$, $(4, 0)$, and $(5, 7)$ are vertices of a right triangle.

Solution

The three points are plotted in Figure P.13. Using the Distance Formula, you can find the lengths of the three sides as follows.

$$d_1 = \sqrt{(5-2)^2 + (7-1)^2} = \sqrt{9 + 36} = \sqrt{45}$$

$$d_2 = \sqrt{(4-2)^2 + (0-1)^2} = \sqrt{4 + 1} = \sqrt{5}$$

$$d_3 = \sqrt{(5-4)^2 + (7-0)^2} = \sqrt{1 + 49} = \sqrt{50}$$

Because

$$d_1^2 + d_2^2 = 45 + 5 = 50 = d_3^2,$$

you can conclude that the triangle must be a right triangle.

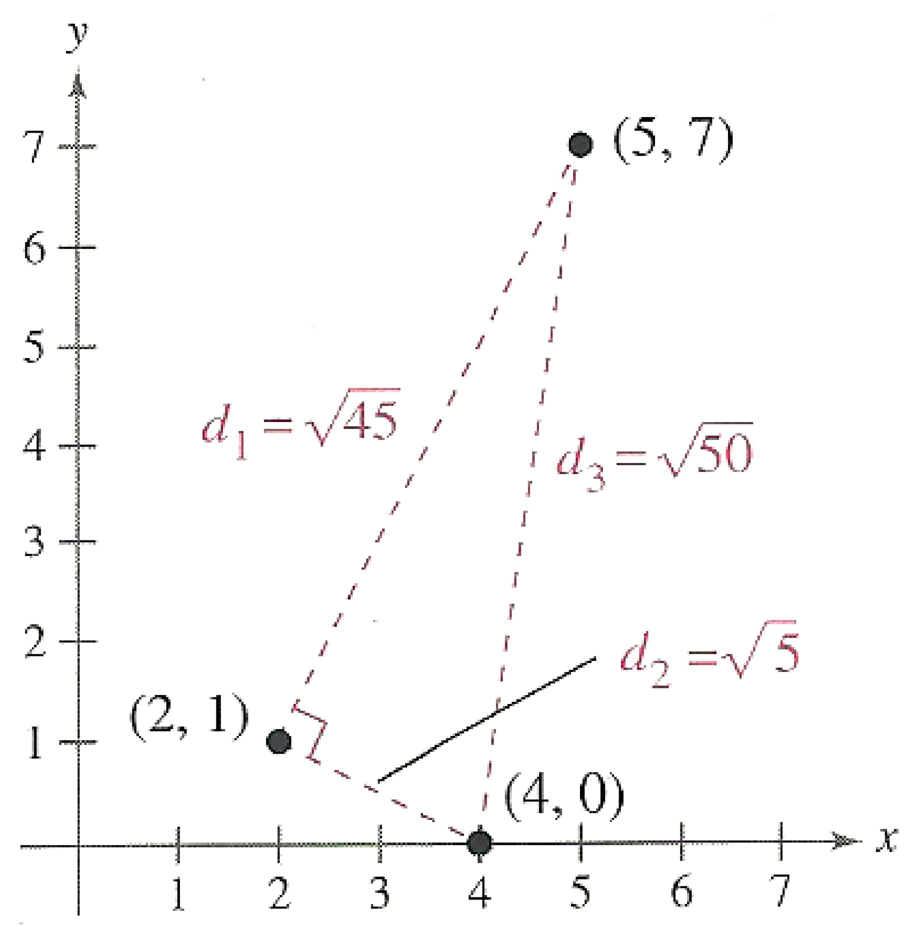


Figure P.13

The figure provided with Example 6 was not really essential to the solution. Nevertheless, it is strongly recommended that you develop the habit of including sketches with your solutions—even if they are not required.

The Midpoint Formula

To find the **midpoint** of the line segment that joins two points in a coordinate plane, you can simply find the average values of the respective coordinates of the two endpoints using the **Midpoint Formula.** (See Appendix A for a proof of the Midpoint Formula.)

The Midpoint Formula

The midpoint of the segment joining the points (x_1, y_1) and (x_2, y_2) is given by the Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

EXAMPLE 7 Finding a Line Segment's Midpoint

Find the midpoint of the line segment joining the points $(-5, -3)$ and $(9, 3)$, as shown in Figure P.14.

Solution

Let $(x_1, y_1) = (-5, -3)$ and $(x_2, y_2) = (9, 3)$.

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \text{Midpoint Formula}$$

$$= \left(\frac{-5 + 9}{2}, \frac{-3 + 3}{2}\right) \quad \text{Substitute for } x_1, y_1, x_2, \text{ and } y_2.$$

$$= (2, 0) \quad \text{Simplify.}$$

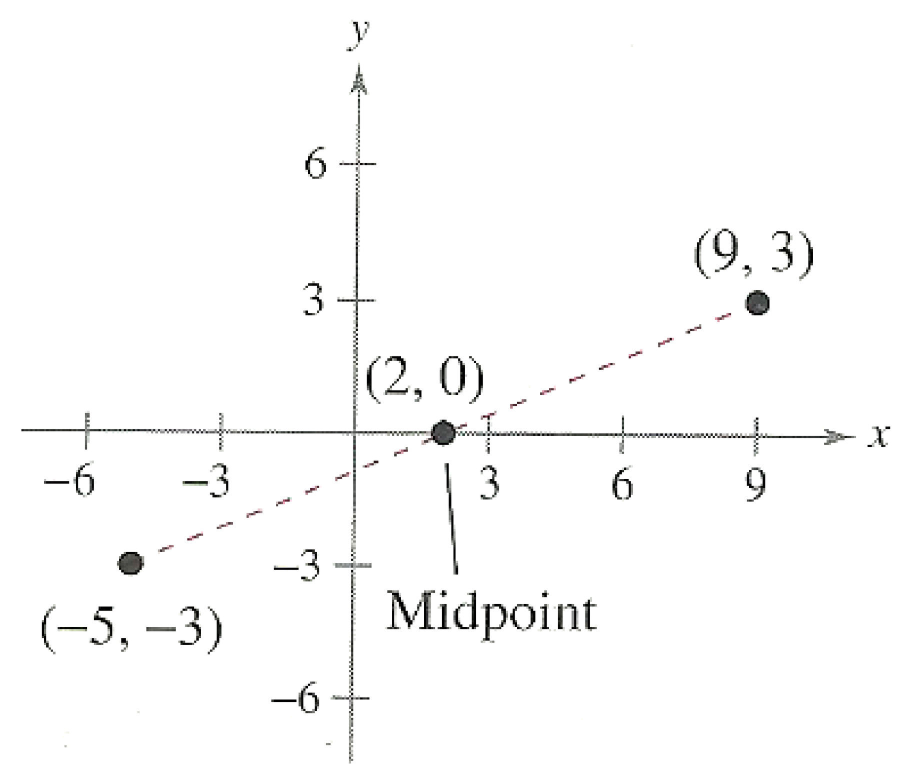


Figure P.14

EXAMPLE 8 Estimating Annual Sales

Winn-Dixie Stores had annual sales of \$1.30 billion in 1996 and \$1.36 billion in 1998. Without knowing any additional information, what would you estimate the 1997 sales to have been? (Source: Winn-Dixie Stores, Inc.)

Solution

One solution to the problem is to assume that sales followed a linear pattern. With this assumption, you can estimate the 1997 sales by finding the midpoint of the segment connecting the points (1996, 1.30) and (1998, 1.36).

$$\text{Midpoint} = \left(\frac{1996 + 1998}{2}, \frac{1.30 + 1.36}{2}\right)$$

$$= (1997, 1.33)$$

So, you would estimate the 1997 sales to have been about \$1.33 billion, as shown in Figure P.15 (the actual 1997 sales were \$1.32 billion).

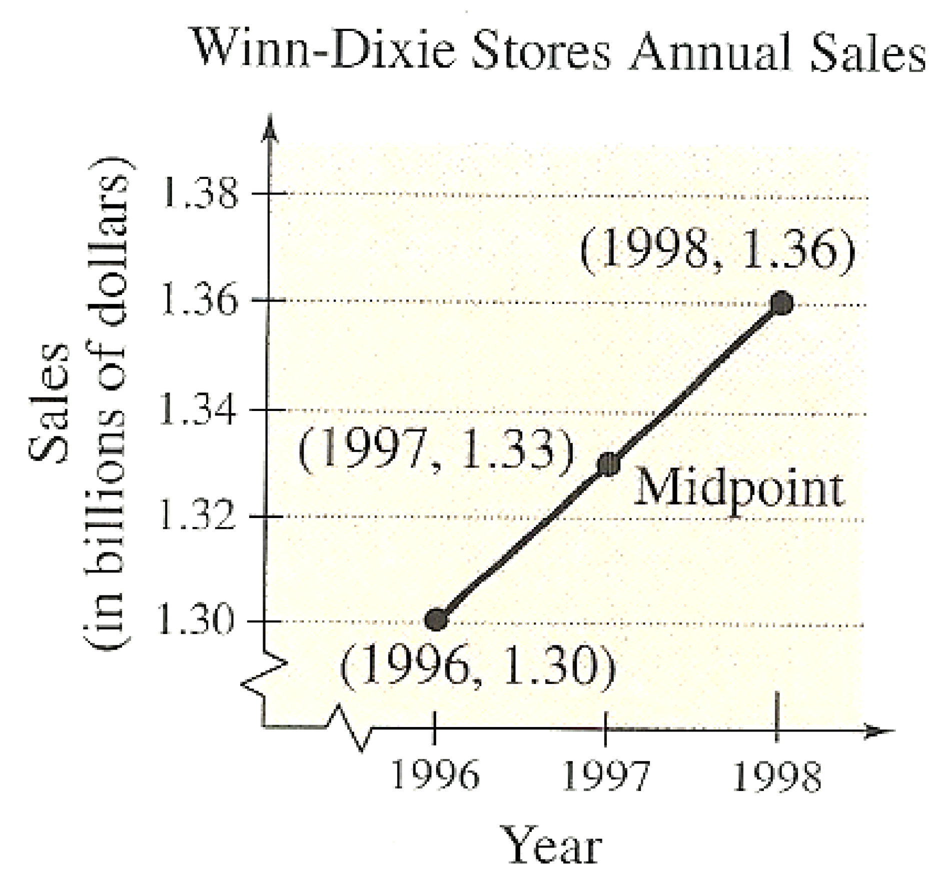


Figure P.15

The Equation of a Circle

The Distance Formula provides a convenient way to define circles. A **circle of radius r** with center at the point (h, k) is shown in Figure P.16. The point (x, y) is on this circle if and only if its distance from the center (h, k) is r. This means that a **circle** in the plane consists of all points (x, y) that are a given positive distance r from a fixed point (h, k). Using the Distance Formula, you can express this relationship by saying that the point (x, y) lies on the circle if and only if

$$\sqrt{(x-h)^2 + (y-k)^2} = r.$$

By squaring both sides of this equation, you can obtain the **standard form of the equation of a circle.**

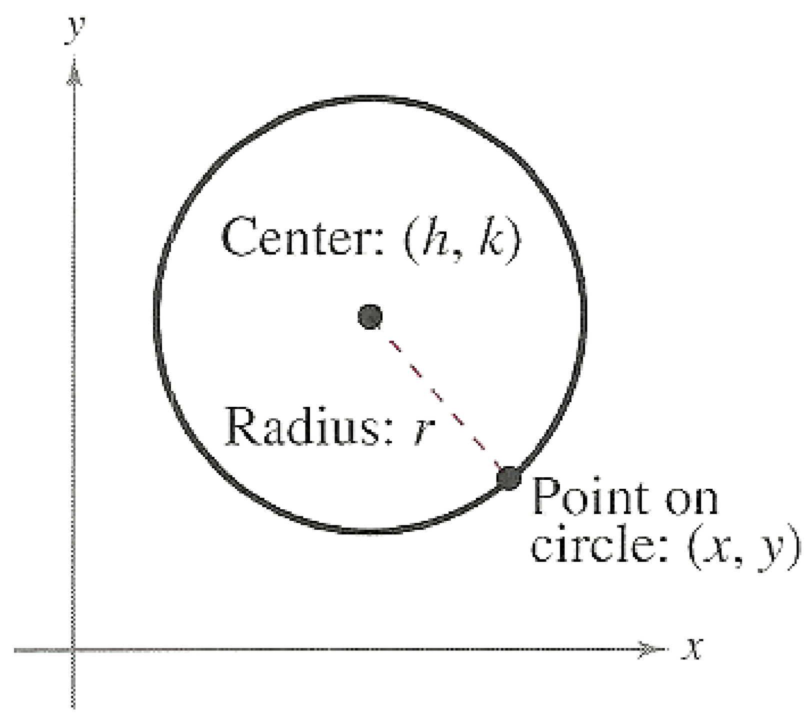


Figure P.16

Standard Form of the Equation of a Circle

The **standard form of the equation of a circle** is

$$(x-h)^2 + (y-k)^2 = r^2.$$

The point (h, k) is the **center** of the circle, and the positive number r is the **radius** of the circle. The standard form of the equation of a circle whose center is the origin is $x^2 + y^2 = r^2$.

EXAMPLE 9 Finding an Equation of a Circle

The point $(3, 4)$ lies on a circle whose center is at $(-1, 2)$, as shown in Figure P.17. Find an equation for the circle.

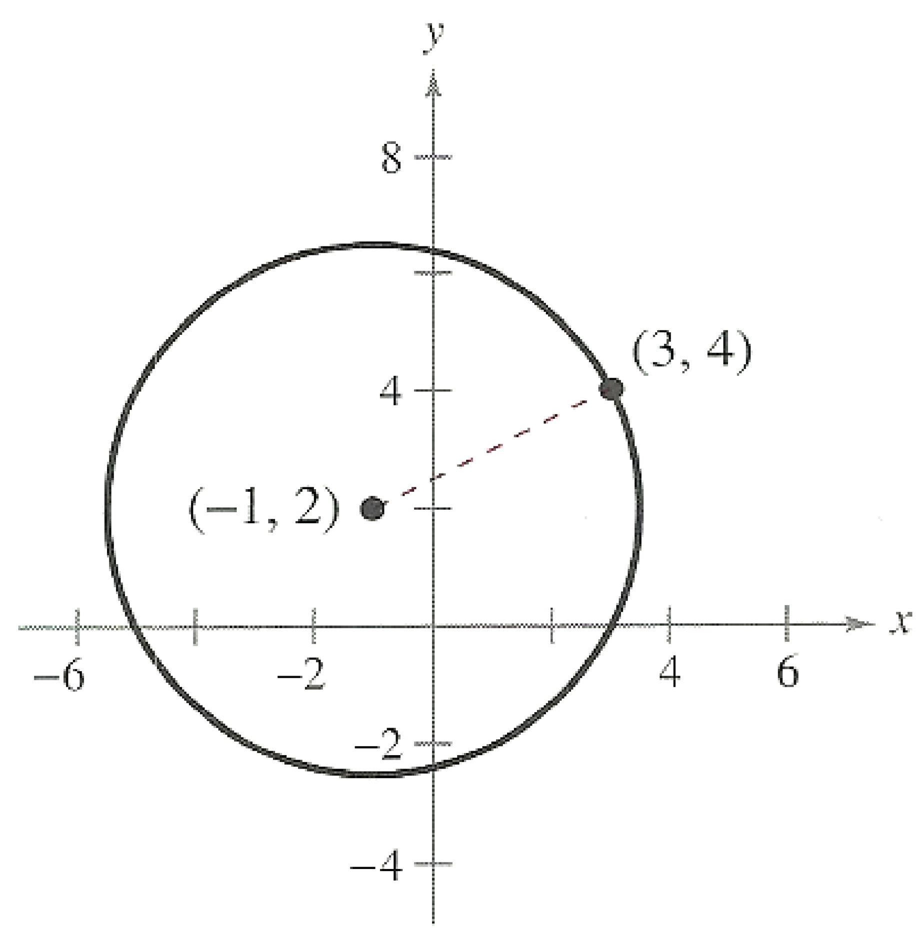


Figure P.17

Solution

The radius r of the circle is the distance between $(-1, 2)$ and $(3, 4)$.

$$r = \sqrt{[3-(-1)]^2 + (4-2)^2}$$
$$= \sqrt{16+4} = \sqrt{20}$$

So, the center of the circle is $(h, k) = (-1, 2)$ and the radius is $r = \sqrt{20}$, and you can write the standard form of the equation of the circle as follows.

$$(x-h)^2 + (y-k)^2 = r^2 \qquad \text{Standard form}$$

$$[x-(-1)]^2 + (y-2)^2 = \left(\sqrt{20}\right)^2 \qquad \text{Substitute for } h, k, \text{ and } r.$$

$$(x+1)^2 + (y-2)^2 = 20 \qquad \text{Equation of circle}$$

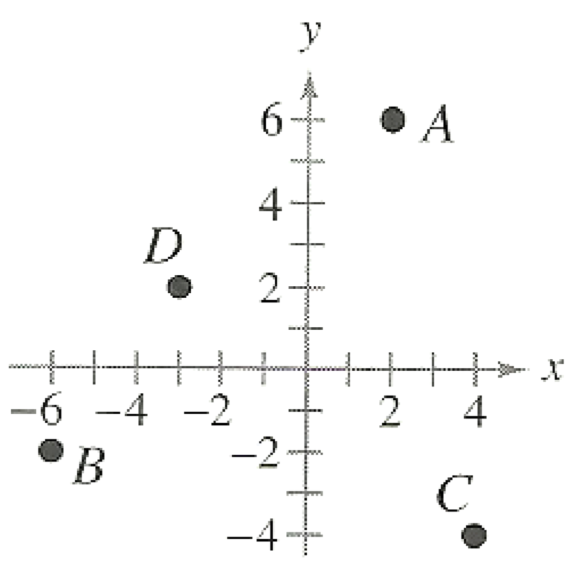
P.1 Exercises

In Exercises 1–4, sketch the polygon with the indicated vertices.

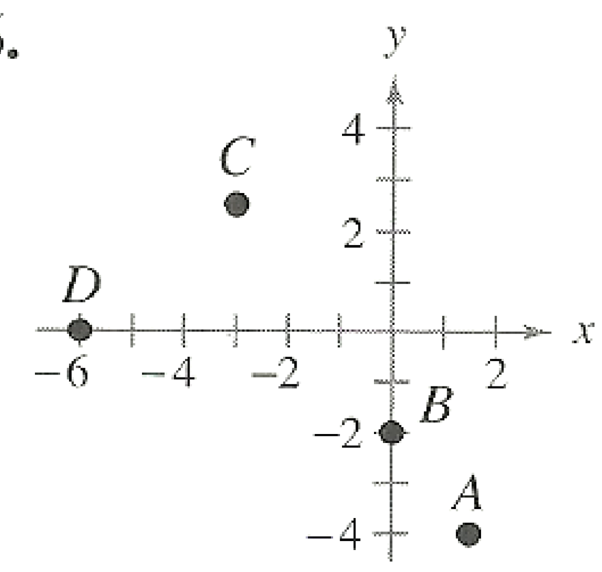
1. Triangle: $(-1, 1), (2, -1), (3, 4)$
2. Triangle: $(0, 3), (-1, -2), (4, 8)$
3. Square: $(2, 4), (5, 1), (2, -2), (-1, 1)$
4. Parallelogram: $(5, 2), (7, 0), (1, -2), (-1, 0)$

In Exercises 5–8, approximate the coordinates of the points.

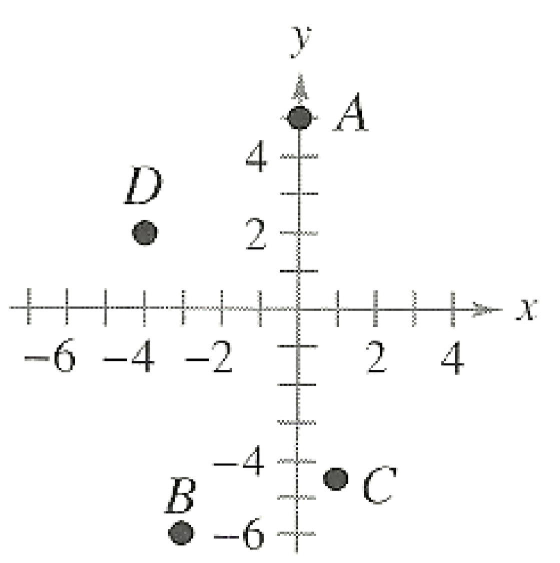
5.



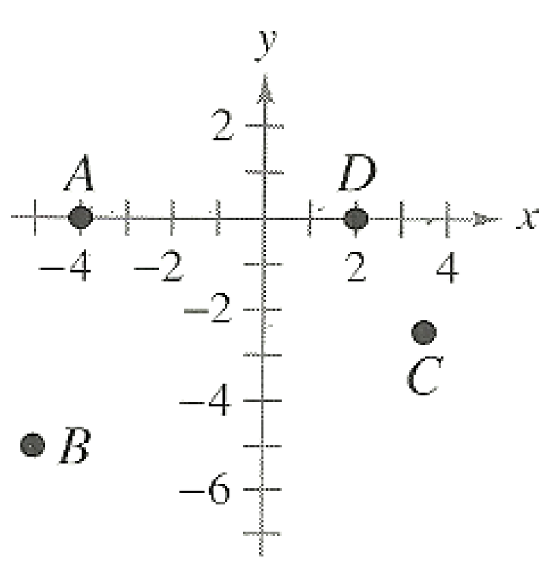
6.



7.



8.



In Exercises 9–12, find the coordinates of the point.

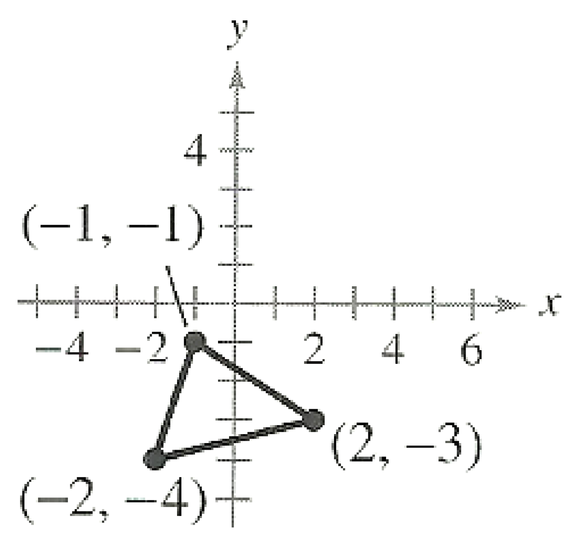
9. The point is located three units to the left of the y-axis and four units above the x-axis.
10. The point is located eight units below the x-axis and four units to the right of the y-axis.
11. The point is located five units below the x-axis and the coordinates of the point are equal.
12. The point is on the x-axis and twelve units to the left of the y-axis.

In Exercises 13–22, determine the quadrant(s) in which (x, y) is located so that the condition(s) is (are) satisfied.

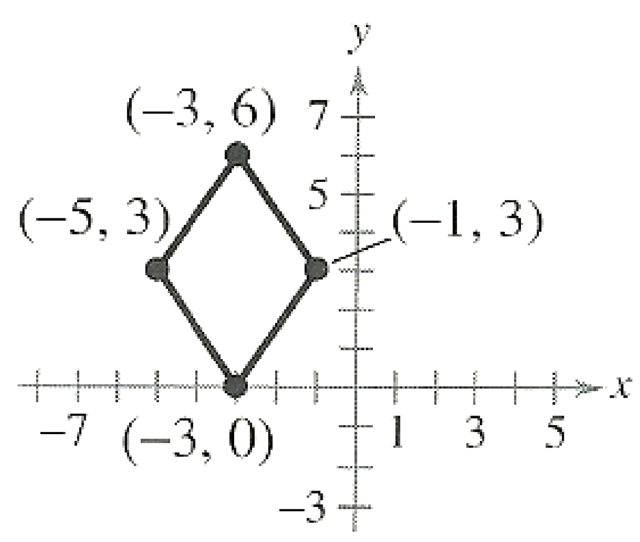
13. $x > 0$ and $y < 0$
14. $x < 0$ and $y < 0$
15. $x = -4$ and $y > 0$
16. $x > 2$ and $y = 3$
17. $y < -5$
18. $x > 4$
19. $(x, -y)$ is in the second quadrant.
20. $(-x, y)$ is in the fourth quadrant.
21. $xy > 0$
22. $xy < 0$

In Exercises 23 and 24, the polygon is shifted to a new position in the plane. Find the coordinates of the vertices of the polygon in its new position.

23. Shift: 5 units up, 2 units to the right



24. Shift: 3 units down, 6 units to the right



In Exercises 25–30, find the distance between the points algebraically and verify graphically by using centimeter graph paper and a centimeter ruler.

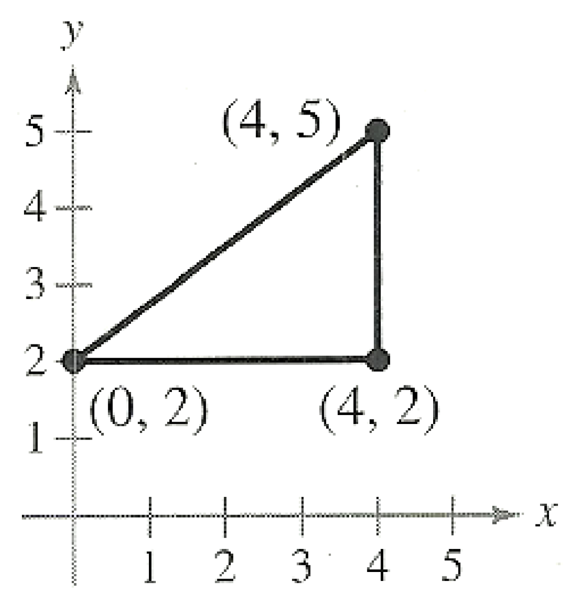
25. $(6, -3), (6, 5)$
26. $(1, 4), (8, 4)$
27. $(-3, -1), (2, -1)$
28. $(-3, -4), (-3, 6)$
29. $(-2, 6), (3, -6)$
30. $(8, 5), (0, 20)$

In Exercises 31–42, (a) plot the points, (b) find the distance between the points, and (c) find the midpoint of the line segment joining the points.

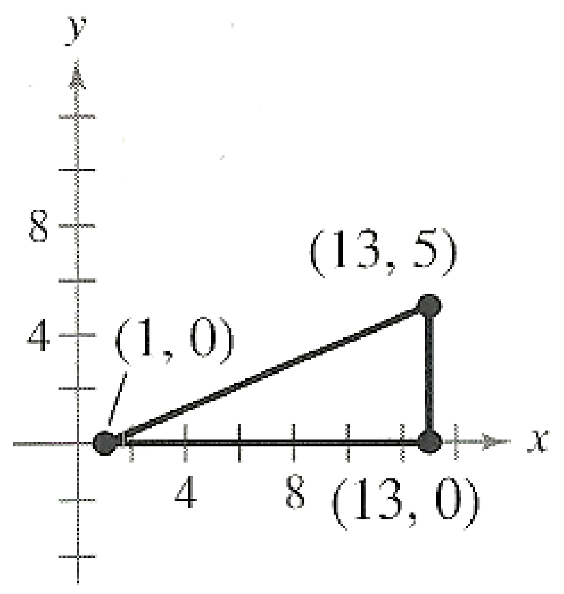
31. $(1, 1), (9, 7)$
32. $(1, 12), (6, 0)$
33. $(-4, 10), (4, -5)$
34. $(-7, -4), (2, 8)$
35. $(-1, 2), (5, 4)$
36. $(2, 10), (10, 2)$
37. $\left(\frac{1}{2}, 1\right), \left(-\frac{5}{2}, \frac{4}{3}\right)$
38. $\left(-\frac{1}{3}, -\frac{1}{3}\right), \left(-\frac{1}{6}, -\frac{1}{2}\right)$
39. $(6.2, 5.4), (-3.7, 1.8)$
40. $(-16.8, 12.3), (5.6, 4.9)$
41. $(-36, -18), (48, -72)$
42. $(1.451, 3.051), (5.906, 11.360)$

In Exercises 43–46, (a) find the length of each side of a right triangle and (b) show that these lengths satisfy the Pythagorean Theorem.

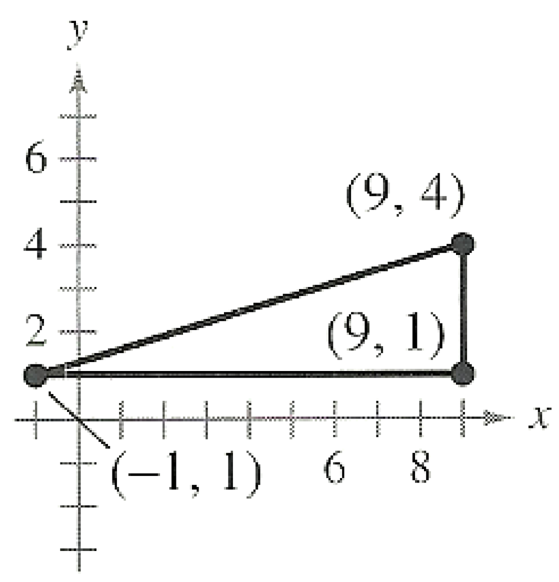
43.



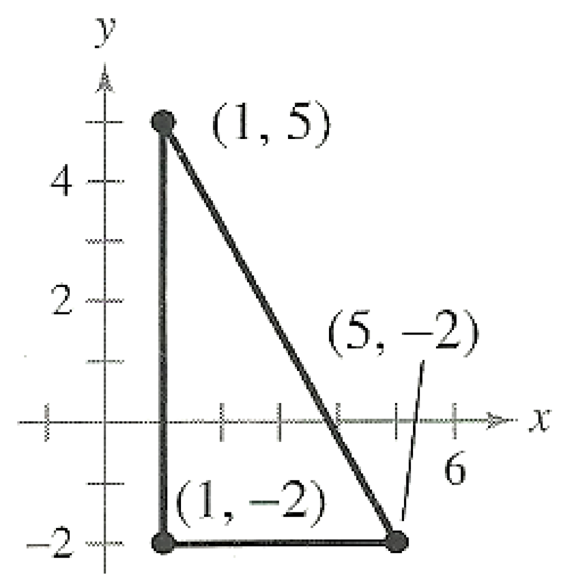
44.



45.



46.



Business **In Exercises 47 and 48, estimate the sales of a company in 1998, given the sales in 1996 and 2000. Assume the sales followed a linear pattern.**

47.

Year	1996	2000
Sales	\$520,000	\$740,000

48.

Year	1996	2000
Sales	\$4,200,000	\$5,650,000

In Exercises 49–52, show that the points form the vertices of the polygon.

49. Right triangle: $(4, 0), (2, 1), (-1, -5)$

50. Isosceles triangle: $(1, -3), (3, 2), (-2, 4)$

51. Parallelogram: $(2, 5), (0, 9), (-2, 0), (0, -4)$

52. Parallelogram: $(0, 1), (3, 7), (4, 4), (1, -2)$

53. ***Exploration*** A line segment has (x_1, y_1) as one endpoint and (x_m, y_m) as its midpoint. Find the other endpoint (x_2, y_2) of the line segment in terms of x_1, y_1, x_m, and y_m. Use the result to find the coordinates of the endpoint of a line segment if the coordinates of the other endpoint and midpoint are, respectively,

(a) $(1, -2), (4, -1)$ (b) $(-5, 11), (2, 4)$.

54. ***Exploration*** Use the Midpoint Formula three times to find the three points that divide the line segment joining (x_1, y_1) and (x_2, y_2) into four parts. Use the result to find the points that divide the line segment joining the given points into four equal parts.

(a) $(1, -2), (4, -1)$ (b) $(-2, -3), (0, 0)$

In Exercises 55–62, find the standard form of the equation of the specified circle.

55. Center: $(0, 0)$; radius: 3

56. Center: $(0, 0)$; radius: 5

57. Center: $(2, -1)$; radius: 4

58. Center: $\left(0, \frac{1}{3}\right)$; radius: $\frac{1}{3}$

59. Center: $(-1, 2)$; solution point: $(0, 0)$

60. Center: $(3, -2)$; solution point: $(-1, 1)$

61. Endpoints of a diameter: $(0, 0), (6, 8)$

62. Endpoints of a diameter: $(-4, -1), (4, 1)$

In Exercises 63–68, find the center and radius, and sketch the circle.

63. $x^2 + y^2 = 4$

64. $x^2 + y^2 = 16$

65. $(x - 1)^2 + (y + 3)^2 = 4$

66. $x^2 + (y - 1)^2 = 4$

67. $\left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = \frac{9}{4}$

68. $\left(x - \frac{2}{3}\right)^2 + \left(y + \frac{1}{4}\right)^2 = \frac{25}{9}$

In Exercises 69 and 70, sketch a scatter plot of the data given in the table.

69. ***Meteorology*** The table shows the lowest temperature of record y (in degrees Fahrenheit) in Duluth, Minnesota, for each month x, where $x = 1$ represents January. (Source: NOAA)

x	1	2	3	4	5	6
y	−39	−33	−29	−5	17	27

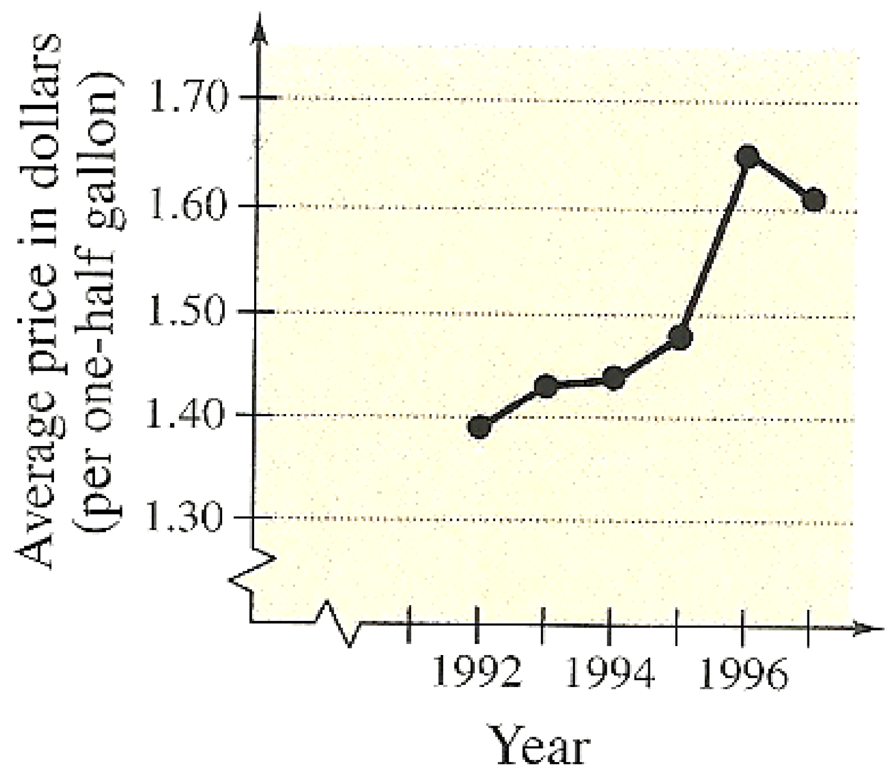
x	7	8	9	10	11	12
y	35	32	22	8	−23	−34

70. ***Business*** The table shows the number y of Wal-Mart stores for each year x from 1992 through 1999. (Source: Wal-Mart Stores, Inc.)

x	1992	1993	1994	1995
y	2136	2440	2759	2943

x	1996	1997	1998	1999
y	3054	3406	3630	3815

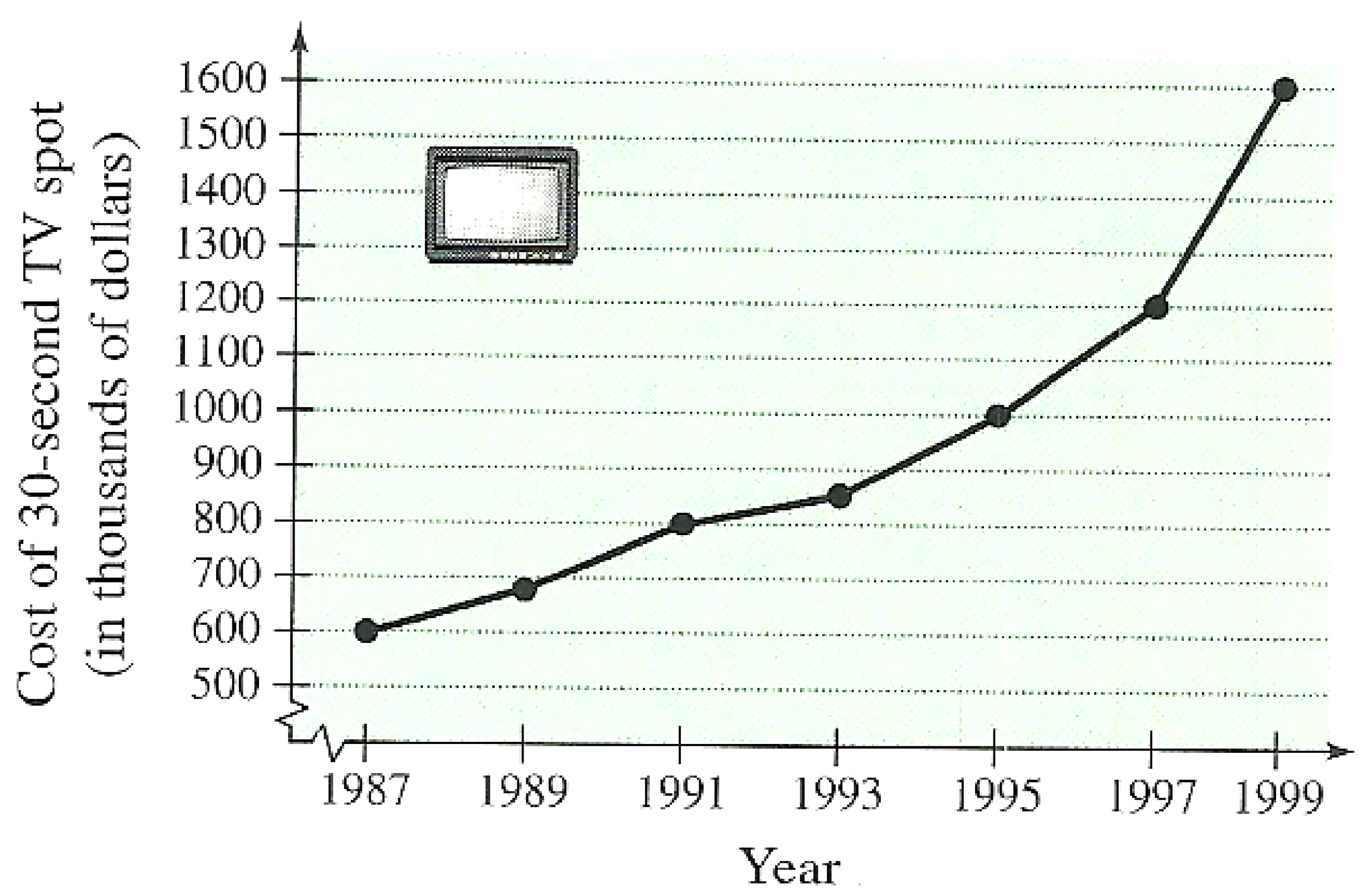
Milk Prices **In Exercises 71 and 72, use the graph below, which shows the average retail price of one-half gallon of milk from 1992 to 1997.** (Source: U.S. Bureau of Labor Statistics)



71. Approximate the highest price of one-half gallon of milk shown in the graph. When did this occur?

72. Approximate the difference in the price of milk from the highest price shown in the graph to the price in 1992.

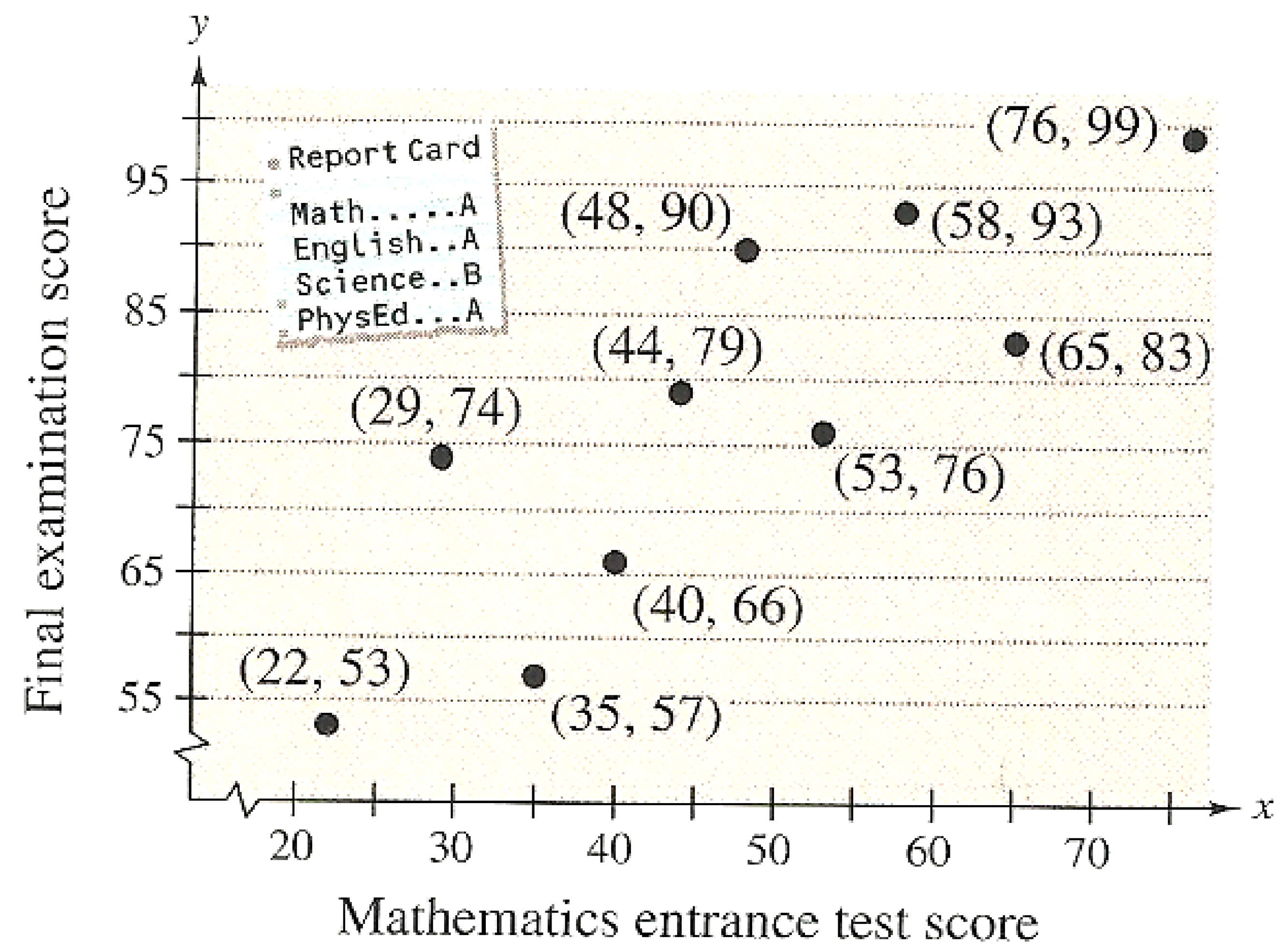
Advertising **In Exercises 73 and 74, use the graph below, which shows the cost of a 30-second television spot (in thousands of dollars) during the Super Bowl from 1987 to 1999.** (Source: USA Today Research)



73. Approximate the percent increase in cost of a 30-second spot from Super Bowl XXI in 1987 to Super Bowl XXXIII in 1999.

74. Estimate the increase in cost of a 30-second spot (a) from Super Bowl XXI to Super Bowl XXVII, and (b) from Super Bowl XXVII to Super Bowl XXXIII.

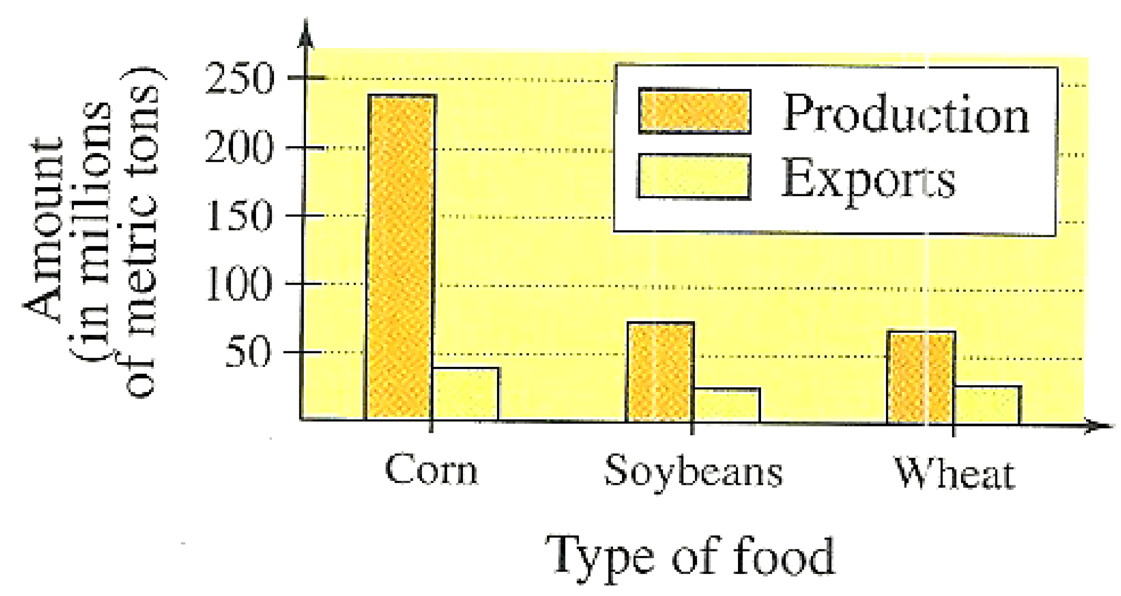
Analyzing Data **In Exercises 75 and 76, refer to the scatter plot, which shows the mathematics entrance test scores x and the final examination scores y in an algebra course for a sample of 10 students.**



75. Find the entrance exam score of any student with a final exam score in the 80s.

76. Does a higher entrance exam score necessarily imply a higher final exam score? Explain.

77. ***Food Production*** The double bar graph shows the production and exports (in millions of metric tons) of corn, soybeans, and wheat for the year 1997. Approximate the percent of each product that is exported. (Source: U.S. Department of Agriculture)



78. ***Fruit Crops*** The table shows farmers' cash receipts (in millions of dollars) from fruit crops in 1996. Construct a bar graph for the data. (Source: U.S. Department of Agriculture)

Fruit	Receipts	Fruit	Receipts
Apples	1846	Oranges	1798
Cherries	264	Peaches	380
Cranberries	246	Pears	292
Grapes	2334	Plums and Prunes	295
Lemons	228	Strawberries	770

79. ***Sports Participants*** The table shows the number of males and females (in millions) over the age of seven that participated in popular sports activities in 1996 in the United States. Construct a double bar graph for the data. (Source: National Sporting Goods Association)

Activity	Male	Female
Aerobic exercising	5.3	18.8
Basketball	22.4	10.9
Bicycling	28.6	24.7
Bowling	22.6	20.3
Camping	24.1	20.6
Exercise walking	26.7	46.6
Running	12.3	9.9
Swimming	29.1	31.1

80. ***Oil Imports*** The table shows the amount of crude oil imported into the United States (in millions of barrels) for the years 1988 through 1997. Construct a line graph for the data and state what information the graph reveals. (Source: Energy Information Administration)

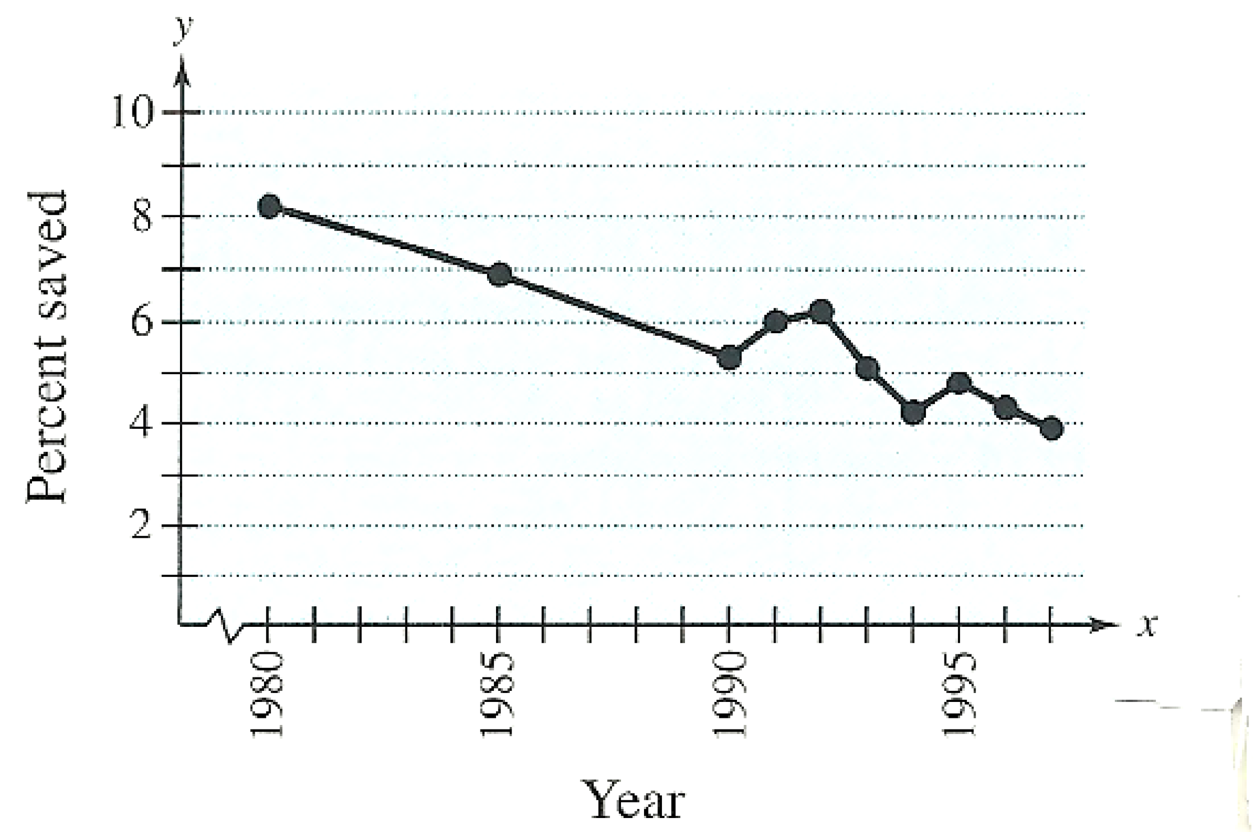
Year	1988	1989	1990	1991	1992
Imports	1864	2133	2151	2110	2220

Year	1993	1994	1995	1996	1997
Imports	2477	2578	2643	2748	2918

81. ***Personal Savings*** The line graph shows the percent of disposable income saved in the United States in selected years from 1980 to 1997. (Source: U.S. Bureau of Economic Analysis)

(a) Determine the percent decrease in the rate of saving from 1980 to 1997.

(b) Is the trend shown in the line graph good for the country? Explain your reasoning.



82. ***Population*** The population P (in millions) of Texas from 1990 to 1997 can be modeled by

$$P = 0.35x + 16.99$$

where x is the time in years, with $x = 0$ corresponding to 1990. (Source: U.S. Bureau of the Census)

(a) Use your graphing utility to create a bar graph of the model for the years 1990 to 1997. Use the graph to determine when the population of Texas exceeded 19 million.

(b) Determine algebraically when the population of Texas exceeded 19 million. Use your graph from part (a) to check your answer.

83. ***Health*** The average patient cost C to a community hospital per day in the United States from 1989 to 1996 can be modeled by

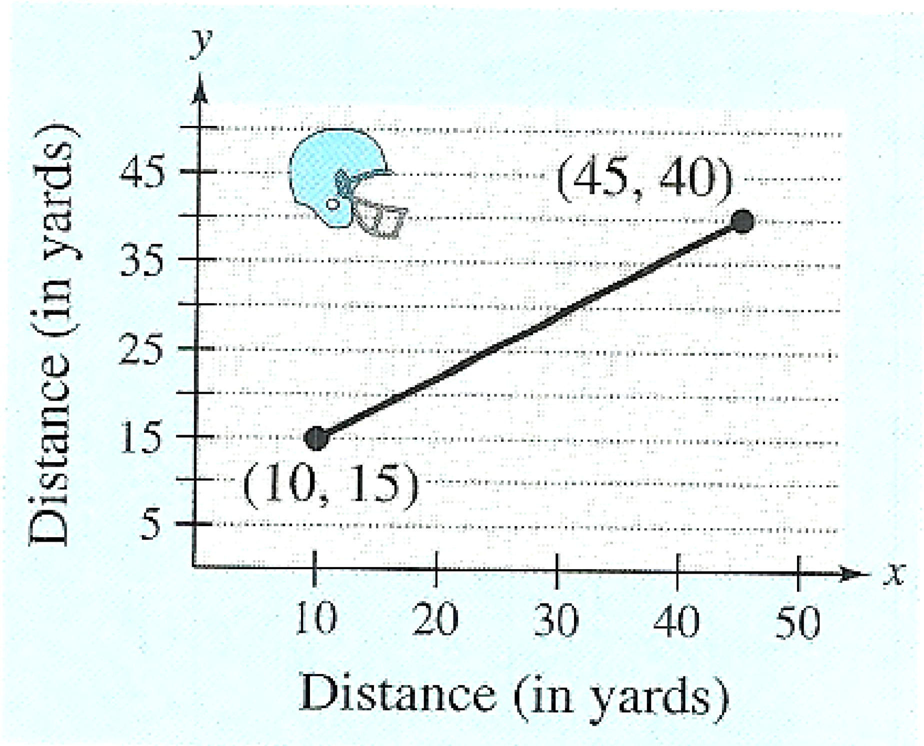
$$C = -2.37t^2 + 66.44t + 696.39$$

where t is the time in years, with $t = 0$ corresponding to 1990. (Source: American Hospital Association)

(a) Determine algebraically when the average cost exceeded \$900 per day.

(b) Check your answer in part (a) by constructing a line graph of the model for the years 1989 to 1996. Use your graph to approximate when the average cost exceeded \$900 per day.

84. ***Sports*** In a football game, a quarterback throws a pass from the 15-yard line, 10 yards from the sideline as shown in the figure. The pass is caught on the 40-yard line, 45 yards from the same sideline. How long is the pass?



85. ***Flying Distance*** A plane flies in a straight line to a city that is 100 kilometers east and 150 kilometers north of the point of departure. How far does it fly?

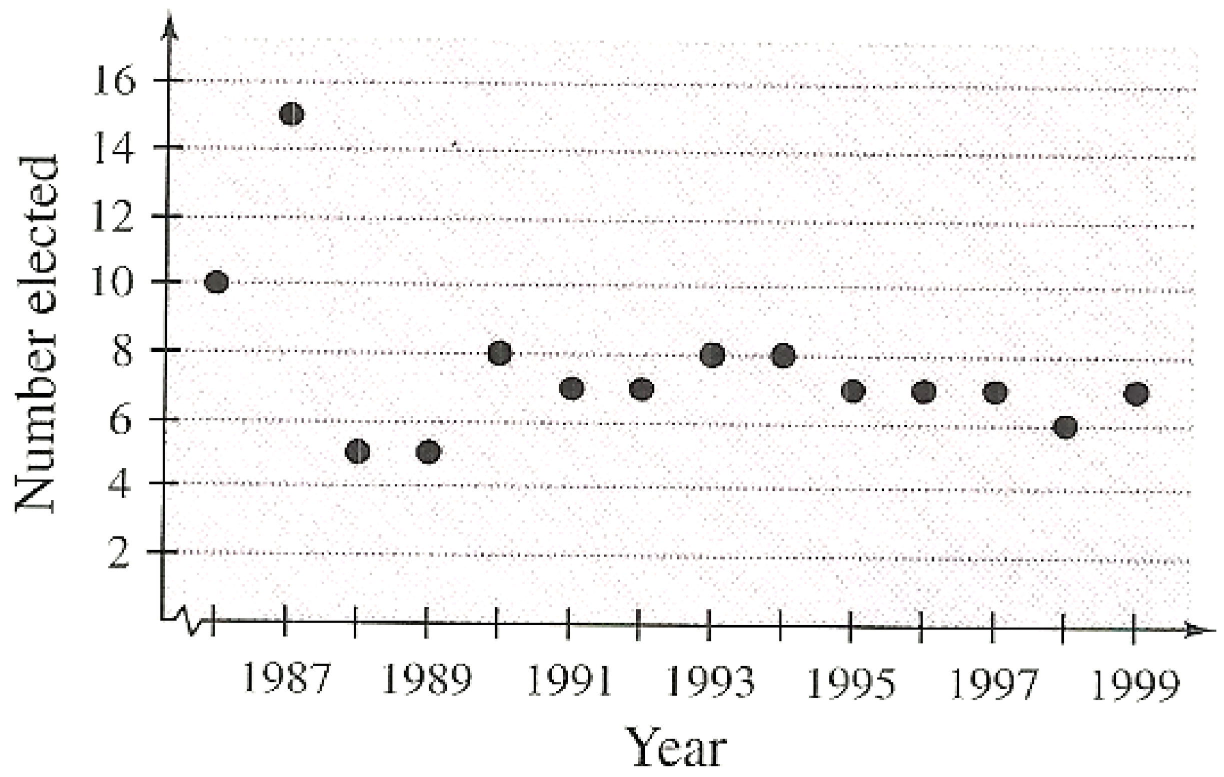
86. ***Make a Conjecture*** Plot the points $(2, 1)$, $(-3, 5)$, and $(7, -3)$ on a rectangular coordinate system. Then change the sign of the indicated coordinate(s) of each point and plot the three new points on the same rectangular coordinate system. Make a conjecture about the location of a point when

(a) the sign of the x-coordinate is changed;

(b) the sign of the y-coordinate is changed;

(c) the sign of both the x- and y-coordinates are changed.

87. ***Rock and Roll Hall of Fame*** The graph below shows the number of recording artists who were elected to the Rock and Roll Hall of Fame from 1986 to 1999.



(a) Describe any trends in the data. From these trends, estimate the number of artists that will be elected in 2001.

(b) Why do you think the numbers elected in 1986 and 1987 were greater than in other years?

88. ***Business*** Starbucks Corporation had annual sales of \$696.5 million in 1996 and \$1308.7 million in 1998. Use the Midpoint Formula to estimate the 1997 sales. (Source: Starbucks Corporation)

89. ***Business*** Lands' End, Inc., had annual sales of \$1118.7 million in 1996 and \$1371.4 million in 1998. Use the Midpoint Formula to estimate the 1997 sales. (Source: Lands' End, Inc.)

Synthesis

True or False? **In Exercises 90–92, determine whether the statement is true or false. Justify your answer.**

90. In order to divide a line segment into 16 equal parts, you would have to use the Midpoint Formula 16 times.

91. The points $(-8, 4)$, $(2, 11)$, and $(-5, 1)$ represent the vertices of an isosceles triangle.

92. If four points represent the vertices of a polygon, and the four sides are equal, then the polygon must be a square.

93. ***Think About It*** What is the y-coordinate of any point on the x-axis? What is the x-coordinate of any point on the y-axis?

94. ***Think About It*** When plotting points on the rectangular coordinate system, is it true that the scales on the x- and y-axes must be the same? Explain.

P.2 Graphs of Equations

The Graph of an Equation

News magazines often show graphs comparing the rate of inflation, the federal deficit, wholesale prices, or the unemployment rate to the time of year. Industrial firms and businesses use graphs to report their monthly production and sales statistics. Such graphs provide geometric pictures of the way one quantity changes with respect to another. Frequently, the relationship between two quantities is expressed as an equation. This section introduces the basic procedure for determining the geometric picture associated with an equation.

For an equation in variables x and y, a point (a, b) is a **solution point** if the substitution of $x = a$ and $y = b$ satisfies the equation. Most equations have *infinitely* many solution points. For example, the equation

$$3x + y = 5$$

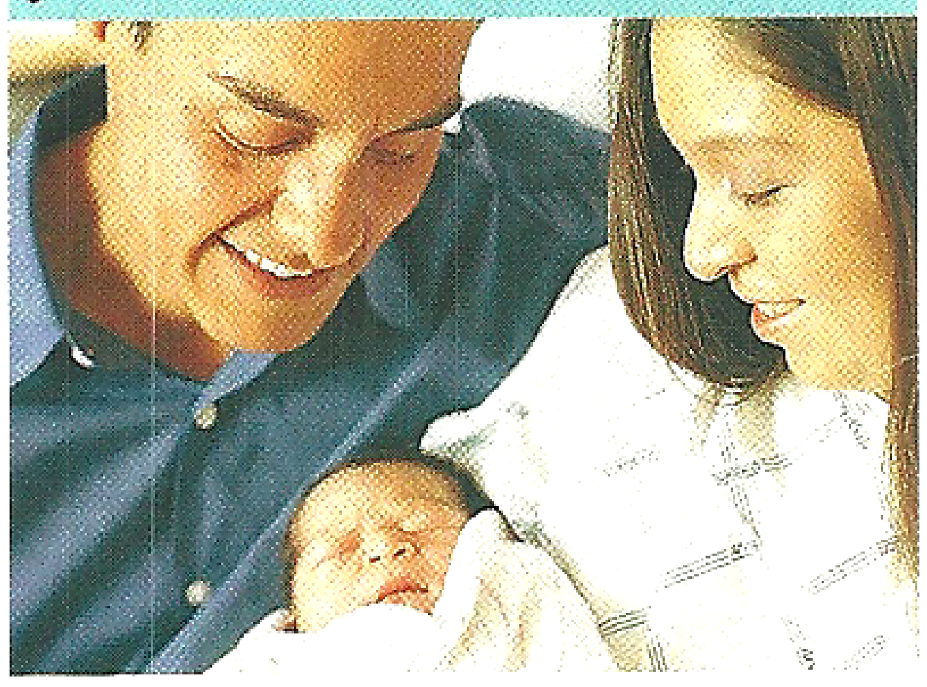
has solution points $(0, 5)$, $(1, 2)$, $(2, -1)$, $(3, -4)$, and so on. The set of all solution points of an equation is the **graph of the equation.**

What You Should Learn:

- How to sketch graphs of equations by point plotting
- How to sketch graphs of equations using a graphing utility
- How to use graphs of equations in real-life problems

Why You Should Learn It:

The graph of an equation can help you see relationships between real-life quantities. For example, Exercise 75 on page 24 shows how a graph can be used to understand the relationship between life expectancy and the year a child is born.



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EXAMPLE 1 Determining Solution Points

Determine whether each point lies on the graph of $y = 10x - 7$.

a. $(2, 13)$ **b.** $(-1, -3)$

Solution

a. The point $(2, 13)$ lies on the graph of $y = 10x - 7$ because it is a solution point of the equation.

$y = 10x - 7$	Write original equation.
$13 \stackrel{?}{=} 10(2) - 7$	Substitute 2 for x and 13 for y.
$13 = 13$	$(2, 13)$ is a solution. ✓

b. The point $(-1, -3)$ does not lie on the graph of $y = 10x - 7$ because it is not a solution point of the equation.

$y = 10x - 7$	Write original equation.
$-3 \stackrel{?}{=} 10(-1) - 7$	Substitute -1 for x and -3 for y.
$-3 \neq -17$	$(-1, -3)$ is not a solution.

How to Sketch the Graph of an Equation by Point Plotting

1. If possible, rewrite the equation so that one of the variables is isolated on one side of the equation.
2. Make a table of several solution points.
3. Plot these points in the coordinate plane.
4. Connect the points with a smooth curve.

EXAMPLE 2 Sketching a Graph by Point Plotting

Use point plotting and graph paper to sketch the graph of

$3x + y = 6.$

Solution

In this case you can isolate the variable y.

$y = 6 - 3x$ Solve equation for y.

Using negative, zero, and positive values for x, you can obtain the following table of values (solution points).

x	-1	0	1	2	3
$y = 6 - 3x$	9	6	3	0	-3

Next, plot these points and connect them, as shown in Figure P.18. It appears that the graph is a straight line. You will study lines extensively in Section P.3.

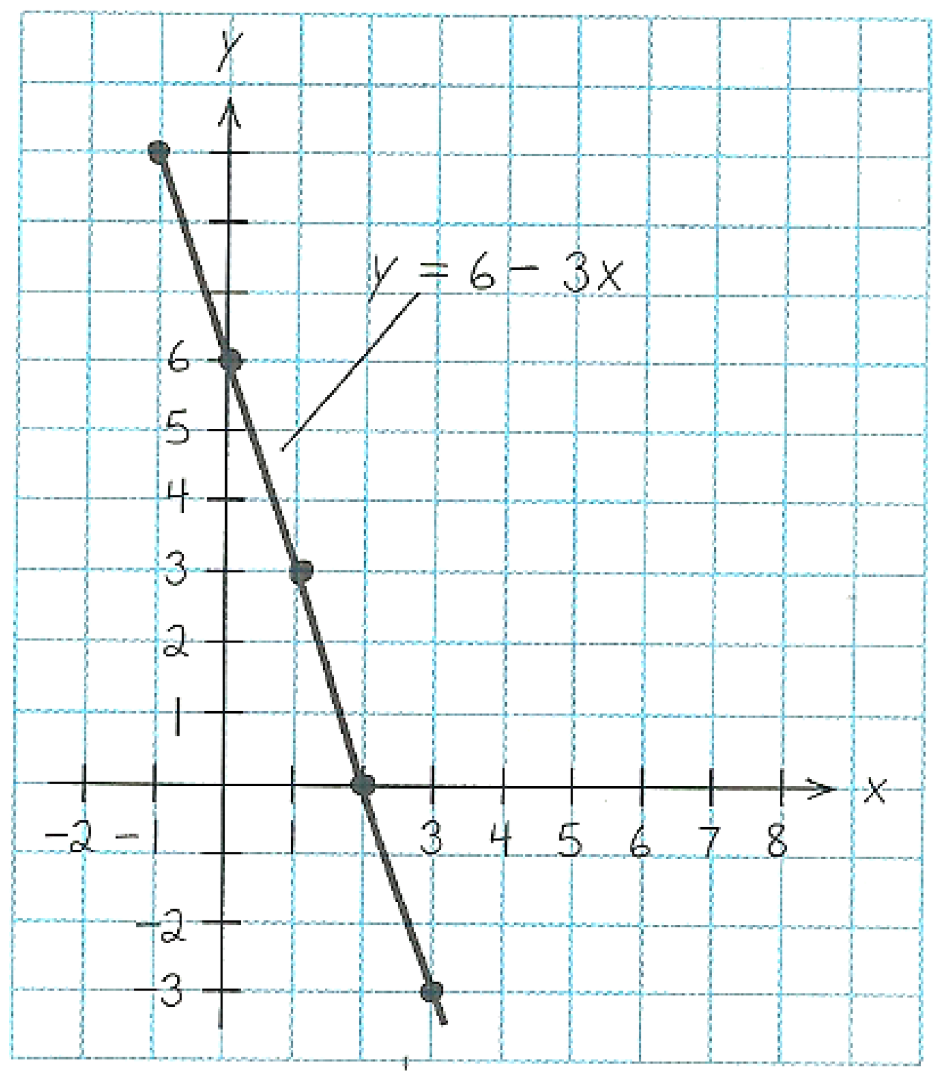


Figure P.18

The points at which a graph touches or crosses an axis are the **intercepts** of the graph. For instance, in Example 2 the point (0, 6) is the y-intercept of the graph because the graph crosses the y-axis at that point. The point (2, 0) is the x-intercept of the graph because the graph crosses the x-axis at that point.

EXAMPLE 3 Sketching a Graph by Point Plotting

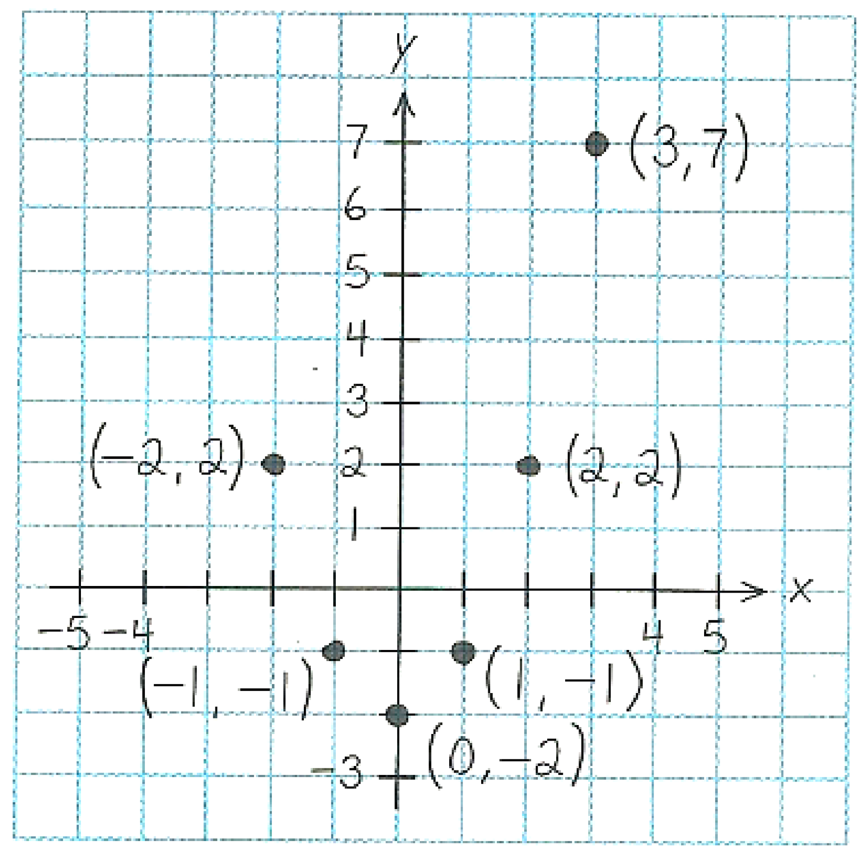
Use point plotting and graph paper to sketch the graph of $y = x^2 - 2$.

Solution

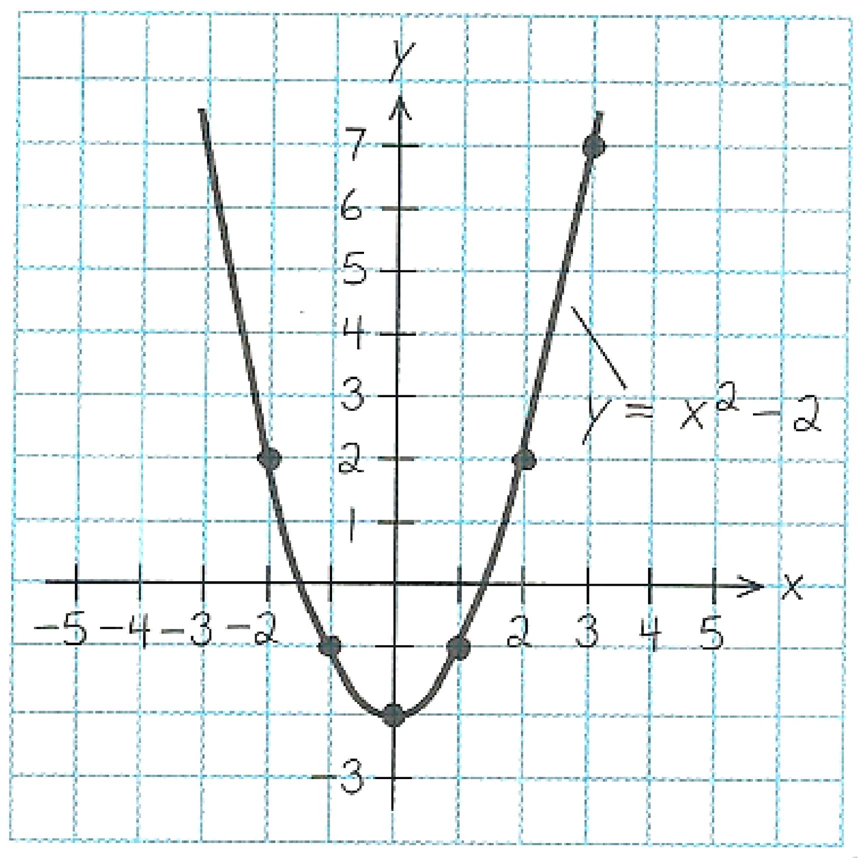
First, make a table of values by choosing several convenient values of x and calculating the corresponding values of y.

x	-2	-1	0	1	2	3
$y = x^2 - 2$	2	-1	-2	-1	2	7

Next, plot the corresponding solution points, as shown in Figure P.19(a). Finally, connect the points with a smooth curve, as shown in Figure P.19(b). This graph is called a *parabola*. You will study parabolas in Section 2.1.



(a)



(b)

Figure P.19

A computer animation of this example appears in the *Interactive* CD-ROM and *Internet* versions of this text.

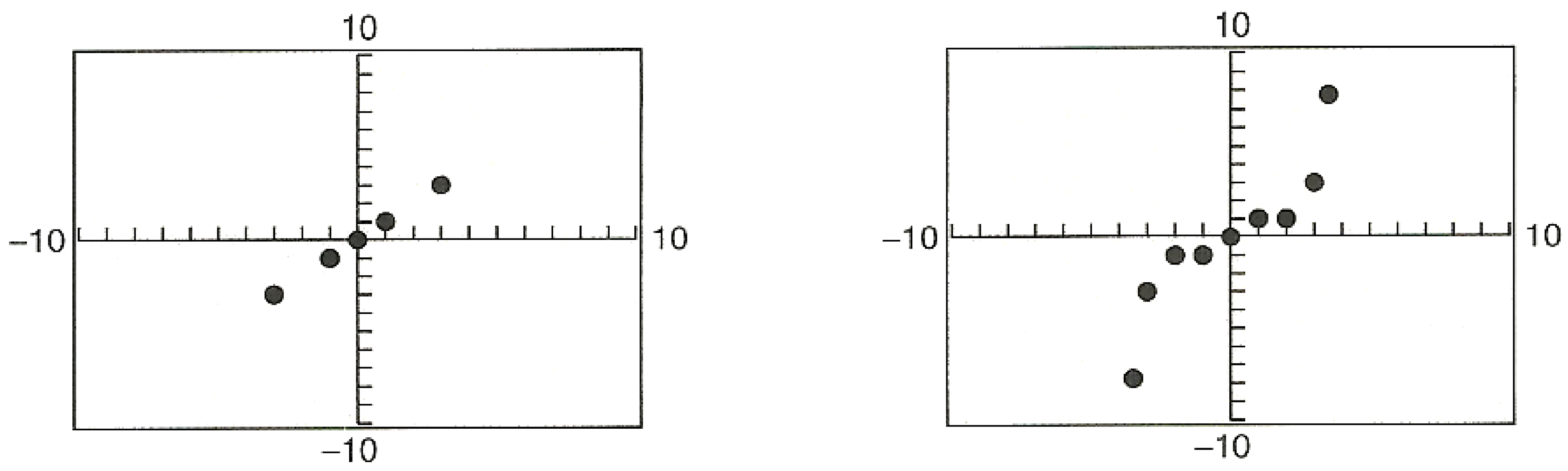
In this text, you will study two basic ways to create graphs: *by hand* and *using a graphing utility*. For instance, the graphs in Figures P.18 and P.19 were sketched by hand and the graph in Figure P.21 was sketched using a graphing utility.

Using a Graphing Utility

One of the disadvantages of the point-plotting method is that to get a good idea about the shape of a graph you need to plot *many* points. With only a few points, you could badly misrepresent the graph. For instance, consider the equation

$$y = \frac{1}{30}x(x^4 - 10x^2 + 39).$$

Suppose you plotted only five points: $(-3, -3)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, and $(3, 3)$, as shown in Figure P.20(a). From these five points, you might assume that the graph of the equation is a straight line. That, however, is not correct. By plotting several more points, you can see that the actual graph is not straight at all, as shown in Figure P.20(b).



(a) (b)

Figure P.20

STUDY TIP

This section presents a brief overview of how to use a graphing utility to graph an equation. For more extensive coverage on this topic, see "An Introduction to Graphing Utilities" on pages xvii–xxii, and the *Graphing Technology Guide.*

From this, you can see that the point-plotting method leaves you with a dilemma. On the one hand, the method can be very inaccurate if only a few points are plotted. But on the other hand, it is very time-consuming to plot a dozen (or more) points. Technology can help solve this dilemma. Plotting several (even several hundred) points on a rectangular coordinate system is something that a computer or calculator can do easily.

The point-plotting method is the method used by *all* graphing utilities. Each computer or calculator screen is made up of a grid of hundreds or thousands of small areas called *pixels*. Screens that have many pixels per square inch are said to have a higher *resolution* than screens with fewer pixels.

Using a Graphing Utility to Graph an Equation

To graph an equation involving x and y on a graphing utility, use the following procedure.

1. Rewrite the equation so that y is isolated on the left side.
2. Enter the equation into a graphing utility.
3. Determine a *viewing window* that shows all important features of the graph.
4. Graph the equation.

EXAMPLE 4 Using a Graphing Utility to Graph an Equation

Use a graphing utility to graph $2y + x^3 = 4x$.

Solution

To begin, solve the equation for y in terms of x.

$$2y + x^3 = 4x \qquad \text{Write original equation.}$$

$$2y = -x^3 + 4x \qquad \text{Subtract } x^3 \text{ from each side.}$$

$$y = -\frac{1}{2}x^3 + 2x \qquad \text{Divide each side by 2.}$$

Now, by entering this equation into a graphing utility (using a standard viewing window), you can obtain the graph shown in Figure P.21.

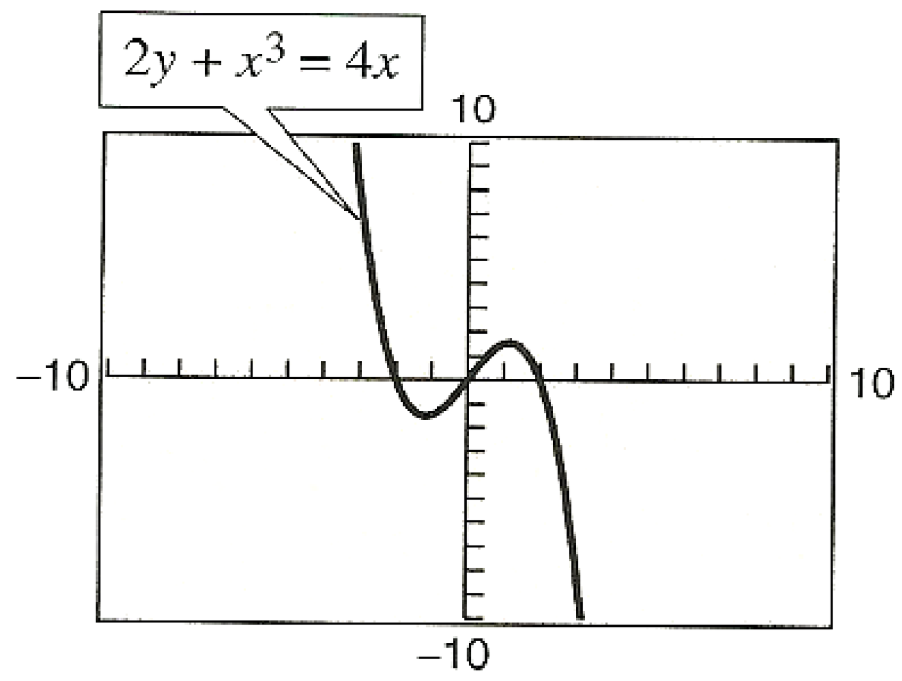


Figure P.21

STUDY TIP

Many graphing utilities have the capability of creating a table, such as the following table, which shows some points of the graph in Figure P.21.

X	Y1
−3	7.5
−2	0
−1	−1.5
0	0
1	1.5
2	0
3	−7.5
X=−3	

By choosing different viewing windows for a graph, it is possible to obtain very different impressions of the graph's shape. For instance, Figure P.22 shows four different viewing windows for the graph of the equation in Example 4. None of these views shows *all* of the important features of the graph as Figure P.21 does.

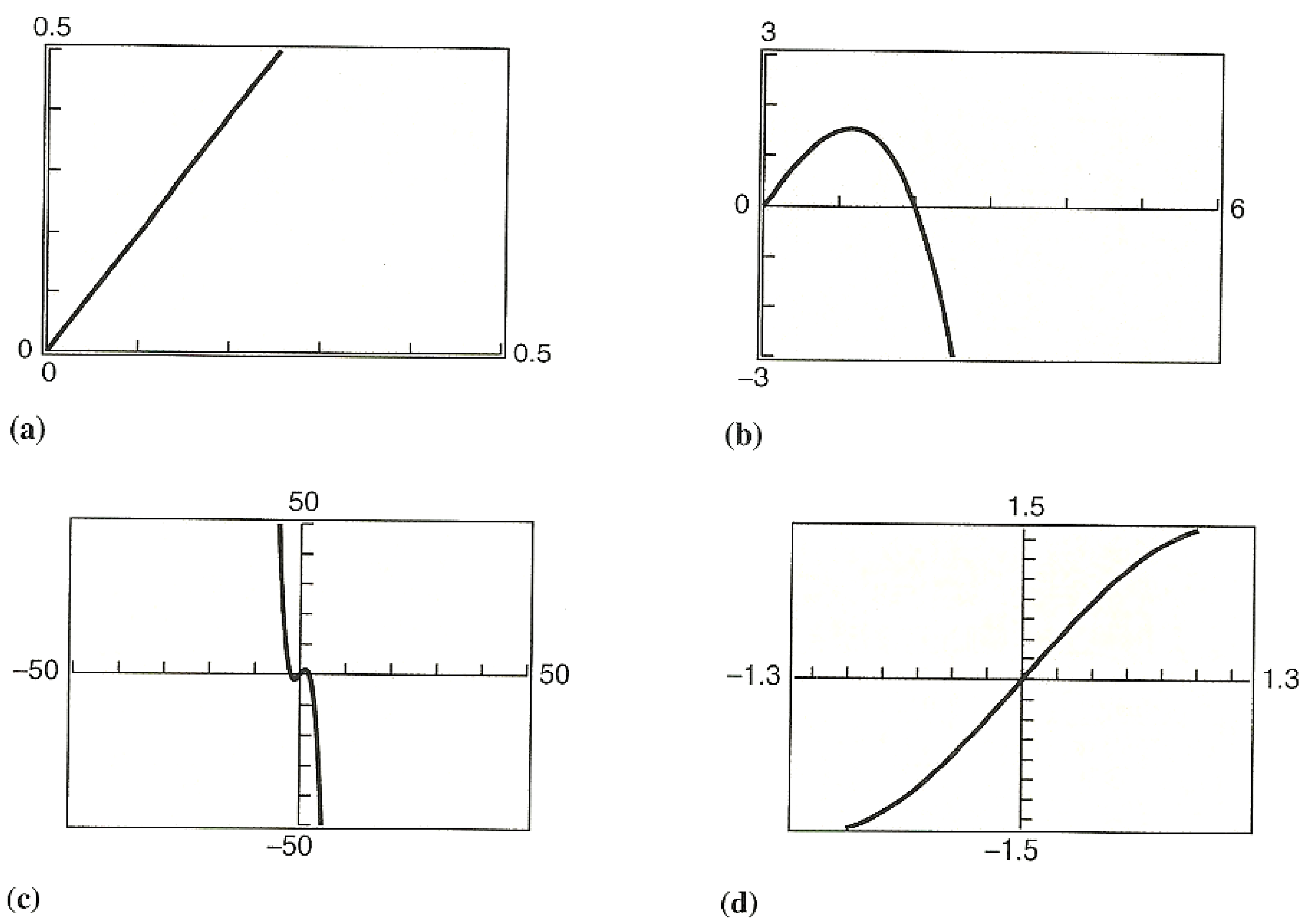


Figure P.22

The standard viewing window on many graphing utilities does not give a true geometric perspective. That is, perpendicular lines will not appear to be perpendicular and circles will not appear to be circular. To overcome this, you can use a *square setting*, as demonstrated in Example 5.

EXAMPLE 5 Sketching a Circle with a Graphing Utility

Use a graphing utility to graph

$$x^2 + y^2 = 9.$$

Solution

The graph of $x^2 + y^2 = 9$ is a circle whose center is the origin and whose radius is 3. (See Section P.1.) To graph the equation, begin by solving the equation for y.

$$x^2 + y^2 = 9 \qquad \text{Write original equation.}$$

$$y^2 = 9 - x^2 \qquad \text{Subtract } x^2 \text{ from each side.}$$

$$y = \pm\sqrt{9 - x^2} \qquad \text{Take square root of each side.}$$

The graph of

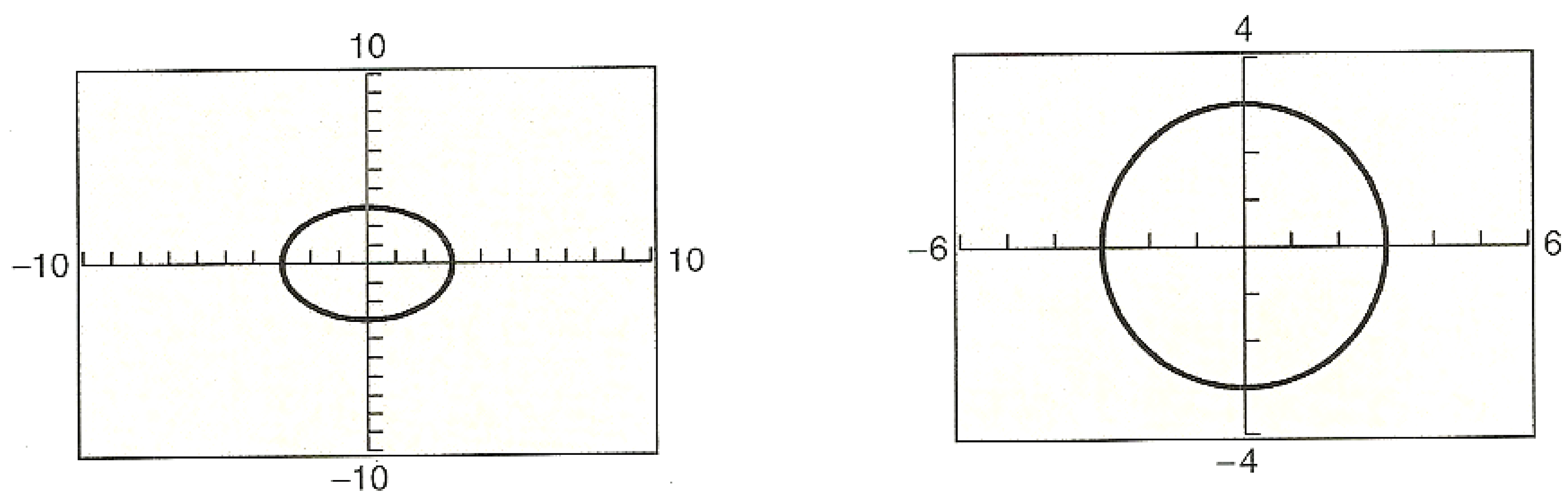
$$y = \sqrt{9 - x^2} \qquad \text{Upper semicircle}$$

is the upper semicircle. The graph of

$$y = -\sqrt{9 - x^2} \qquad \text{Lower semicircle}$$

is the lower semicircle. Enter *both* equations in your graphing utility and generate the resulting graphs. In Figure P.23(a), note that if you use a standard viewing window, the two graphs do not appear to form a circle. You can overcome this problem by using a *square setting*, in which the horizontal and vertical tick marks have equal spacing, as shown in Figure P.23(b). On many graphing utilities, a square setting can be obtained by using a y to x ratio of 2 to 3. For instance, in Figure 1.6(b), the y to x ratio is

$$\frac{\text{Ymax} - \text{Ymin}}{\text{Xmax} - \text{Xmin}} = \frac{4 - (-4)}{6 - (-6)} = \frac{8}{12} = \frac{2}{3}.$$



(a) (b)

Figure P.23

The *Interactive* CD-ROM and *Internet* versions of this text show every example with its solution; clicking on the *Try It!* button brings up similar problems. Guided Examples and Integrated Examples show step-by-step solutions to additional examples. Integrated Examples are related to several concepts in the section.

Throughout this course, you will learn that there are many ways to approach a problem. Two of the three common approaches are illustrated in Example 6.

A Numerical Approach: Construct and use a table.

A Graphical Approach: Draw and use a graph.

An Algebraic Approach: Use the rules of algebra.

We recommend that you habitually use at least two approaches with every problem to help build your intuition and check that your answer is reasonable.

Applications

The following two applications show how to develop mathematical models to represent real-world situations. You will see that both a graphing utility and algebra can be used to understand and solve the problems posed.

STUDY TIP

In applications, it is convenient to use variable names that suggest real-life quantities: d for distance, t for time, and so on. Most graphing utilities, however, require the variable names to be x and y.

EXAMPLE 6 Running a Marathon

A runner runs at a constant rate of 4.9 miles per hour. The verbal model and algebraic equation relating distance run and elapsed time are as follows.

Verbal Model: Distance = Rate · Time *Equation:* $d = 4.9t$

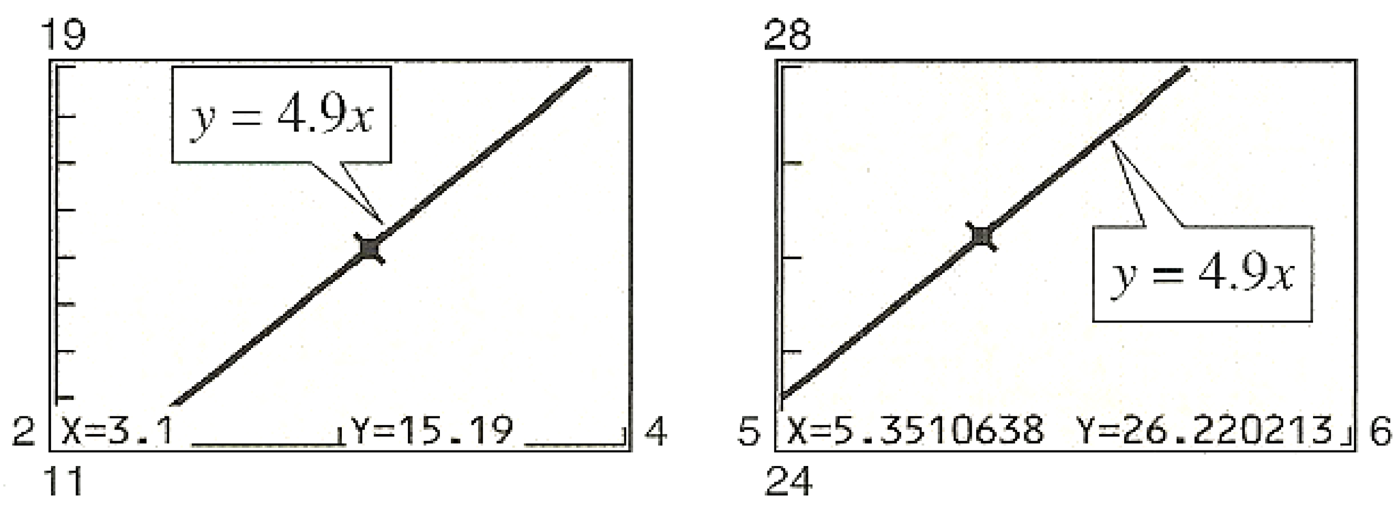
a. Determine how far the runner can run in 3.1 hours.

b. Determine how long it will take to run a 26.2-mile marathon.

Graphical Solution

a. To begin, use a graphing utility to graph the equation $d = 4.9t$. (Represent d by y and t by x.) Be sure to use a viewing window that shows the graph when $x = 3.1$. Then use the *value* feature or *zoom* and *trace* features of the graphing utility to estimate that when $x = 3.1$, the distance is $y \approx 15.2$ miles, as shown in Figure P.24(a).

b. Adjust the viewing window so that it shows the graph when $y = 26.2$. Use the *value* feature or *zoom* and *trace* features to estimate that when $y = 26.2$, the time is $x \approx 5.4$ hours, as shown in Figure P.24(b).



(a) (b)

Figure P.24

Note that the viewing window on your graphing utility may differ slightly from those shown in Figure P.24.

Algebraic Solution

a. To find how far the runner can run in 3.1 hours, substitute 3.1 for t in the equation.

$d = 4.9t$ Write original equation.

$= 4.9(3.1)$ Substitute 3.1 for t.

≈ 15.2 Use a calculator.

So, a runner can run about 15.2 miles in 3.1 hours. Use estimation to check your answer. Because 4.9 is about 5 and 3.1 is about 3, the distance is about $5(3) = 15$. So 15.2 is reasonable.

b. You can find how long it will take to run a 26.2-mile marathon as follows. (For help with solving linear equations see Appendix C.)

$d = 4.9t$ Write original equation.

$\frac{d}{4.9} = t$ Divide each side by 4.9.

$\frac{26.2}{4.9} = t$ Substitute 26.2 for d.

$5.3 \approx t$ Use a calculator.

So, it will take about 5.3 hours to run 26.2 miles.

EXAMPLE 7 · Monthly Wages

You receive a monthly salary of \$2000 plus a commission of 10% of sales. The verbal model and algebraic equations relating the wages, the salary, and the commission are as follows.

Verbal Model: Wages = Salary + Commission on Sales *Equation:* $y = 2000 + 0.1x$

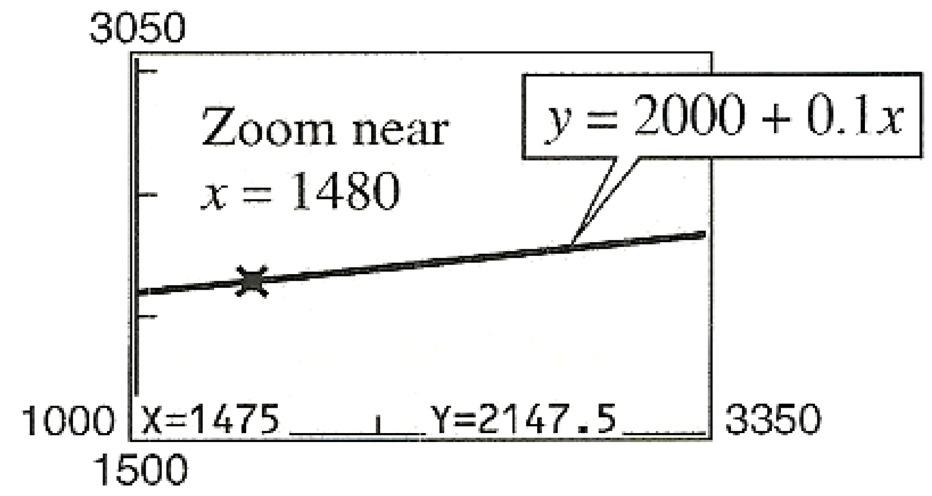
a. If sales are $x = 1480$ in August, what are your wages for that month?

b. If you receive \$2225 for September, what are your sales for that month?

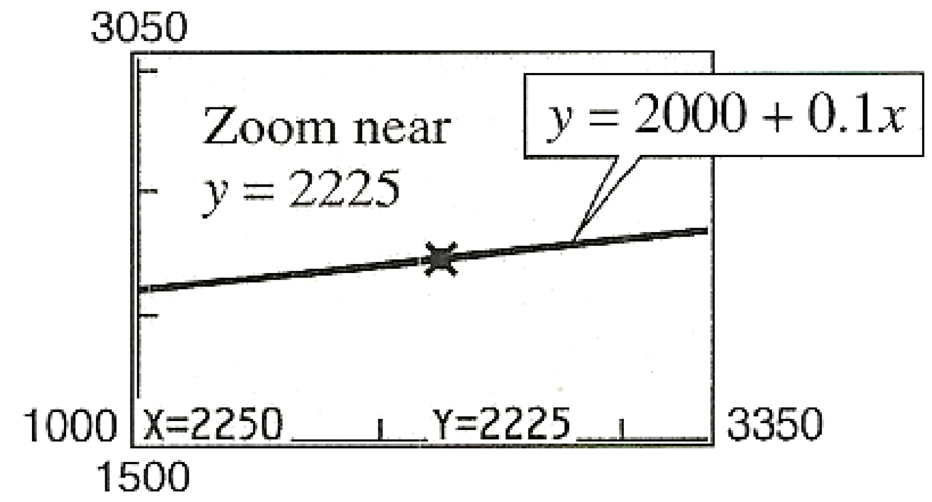
Graphical Solution

a. You can use a graphing utility to graph $y = 2000 + 0.1x$ and then estimate the wages when $x = 1480$. Because $x \geq 0$ and the monthly wages are at least \$2000, a reasonable viewing window is the one shown in Figure P.25(a). Using the *value* feature or *zoom* and *trace* features near $x = 1480$ shows that the wages are about \$2148.

b. Use the graphing utility to find the value along the x-axis (sales) that corresponds to a y-value of 2225 (wages). Beginning with Figure P.25(b) and using the *value* feature or *zoom* and *trace* features, you can estimate the sales to be about \$2250.



(a)



(b)

Figure P.25

Numerical Solution

a. To find the wages in August, evaluate the equation when $x = 1480$.

$$y = 2000 + 0.1x \quad \text{Write original equation.}$$
$$= 2000 + 0.1(1480) \quad \text{Substitute 1480 for } x.$$
$$= 2148 \quad \text{Simplify.}$$

So, the wages in August are \$2148.

b. The table shows the wages for different amounts of sales.

Sales x	2000	2100	2200	2300	2400
Wages y	2200	2210	2220	2230	2240

From the table, you can see that wages of \$2225 result from sales between \$2200 and \$2300. You can improve this estimate by making a table similar to the one below.

Sales x	2210	2220	2230	2240	2250
Wages y	2221	2222	2223	2224	2225

Sales x	2260	2270	2280	2290
Wages y	2226	2227	2228	2229

From the table, you can see that wages of \$2225 result from sales of \$2250.

Writing About Math *Comparison of Wages*

Your employer offers you a choice of wage scales: a monthly salary of \$3000 plus commission of 7% of sales or a salary of \$3400 plus a 5% commission. Write a short paragraph discussing how you would choose your option. At what sales level would the options yield the same salary? Be sure to take advantage of your graphing utility to make your decision.

P.2 Exercises

In Exercises 1–8, determine whether the points lie on the graph of the equation.

	Equation	*Points*	
1.	$y = \sqrt{x + 4}$	(a) $(0, 2)$	(b) $(5, 3)$
2.	$y = x^2 - 3x + 2$	(a) $(2, 0)$	(b) $(-2, 8)$
3.	$y = 4 - \|x - 2\|$	(a) $(1, 5)$	(b) $(1.2, 3.2)$
4.	$y = \dfrac{1}{x^2 + 1}$	(a) $(0, 0)$	(b) $(3, 0.1)$
5.	$2x - y - 3 = 0$	(a) $(1, 2)$	(b) $(1, -1)$
6.	$x^2 + y^2 = 20$	(a) $(3, -2)$	(b) $(-4, 2)$
7.	$x^2y - x^2 + 4y = 0$	(a) $\left(1, \frac{1}{5}\right)$	(b) $\left(2, \frac{1}{2}\right)$
8.	$y = \frac{1}{3}x^3 - 2x^2$	(a) $\left(2, -\frac{16}{3}\right)$	(b) $(-3, 9)$

In Exercises 9–14, complete the table. Use the resulting solution points to sketch the graph of the equation. Use a graphing utility to verify the graph.

9. $y = -2x + 3$

x	-1	0	1	$\frac{3}{2}$	2
y					

10. $y = \frac{3}{2}x - 1$

x	-2	0	$\frac{2}{3}$	1	2
y					

11. $y = x^2 - 2x$

x	-1	0	1	2	3
y					

12. $y = 4 - x^2$

x	-2	-1	0	1	2
y					

13. $y = 3 - |x - 2|$

x	0	1	2	3	4
y					

14. $y = \sqrt{x - 1}$

x	1	2	5	10	17
y					

15. ***Exploration***

(a) Complete the table for the equation $y = \frac{1}{4}x - 3$.

x	-2	-1	0	1	2
y					

(b) Use the resulting solution points to sketch its graph. Then use a graphing utility to verify the graph.

(c) Repeat parts (a) and (b) for the equation $y = -\frac{1}{4}x - 3$. Use the result to describe any differences between the graphs.

16. ***Exploration***

(a) Complete the table for the equation $y = \dfrac{6x}{x^{-2} + 1}$.

x	-2	-1	0	1	2
y					

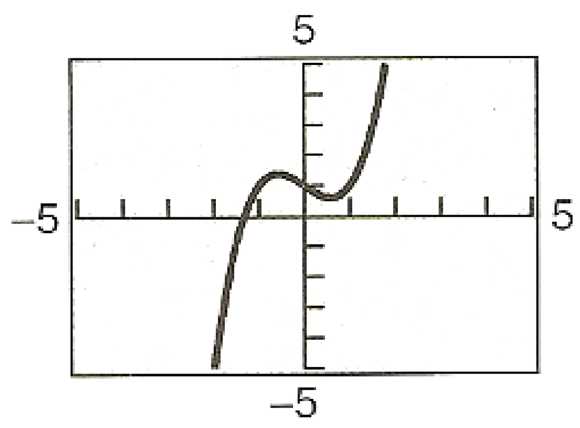
(b) Use the resulting solution points to sketch its graph. Then use a graphing utility to verify the graph.

(c) Continue the table in part (a) for x-values of 5, 10, 20, and 40. What is the value of y approaching? Can y be negative for positive values of x? Explain.

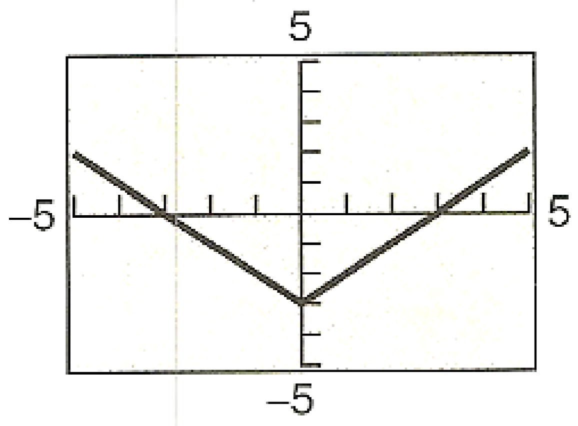
The *Interactive* CD-ROM and *Internet* versions of this text contain step-by-step solutions to all odd-numbered Section and Review Exercises. They also provide Tutorial Exercises, which link to Guided Examples for additional help.

In Exercises 17–22, match the equation with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

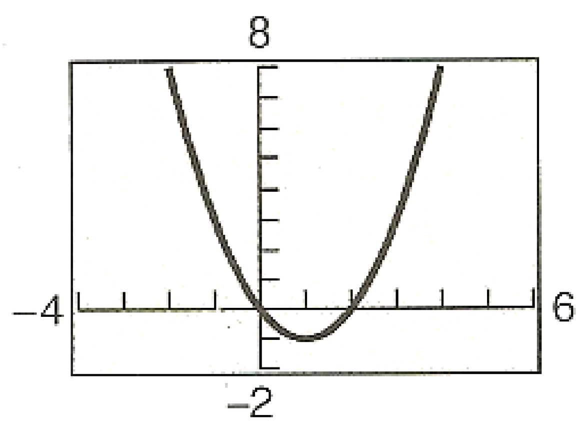
(a)



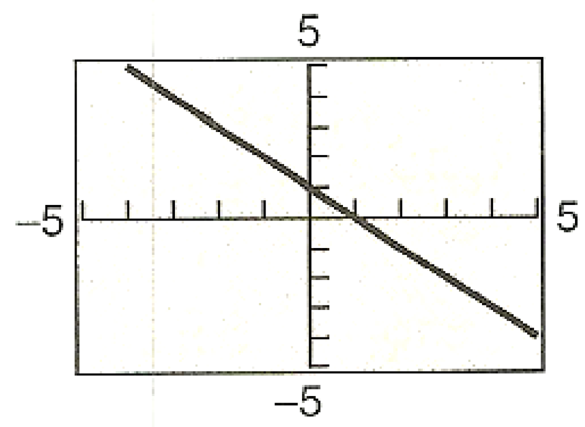
(b)



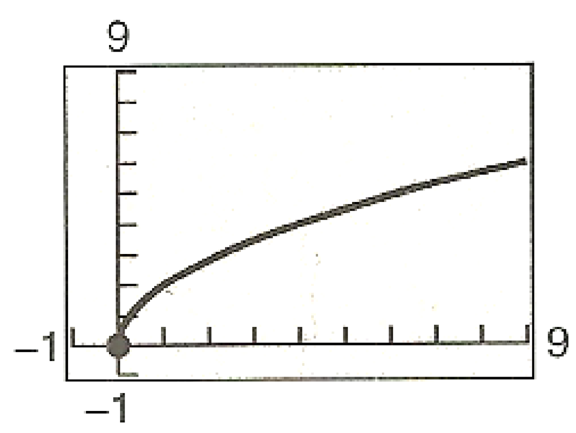
(c)



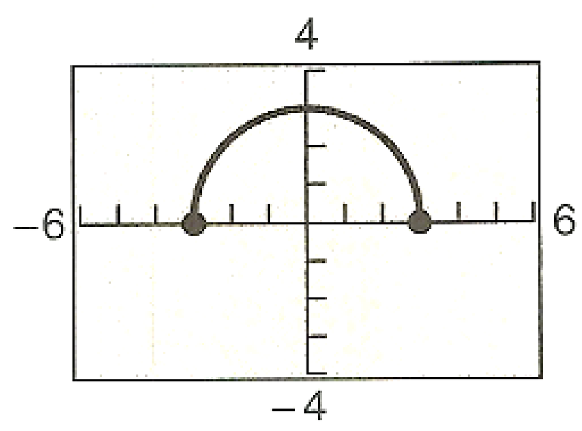
(d)



(e)



(f)



17. $y = 1 - x$ **18.** $y = x^2 - 2x$

19. $y = \sqrt{9 - x^2}$ **20.** $y = 2\sqrt{x}$

21. $y = x^3 - x + 1$ **22.** $y = |x| - 3$

In Exercises 23–36, sketch the graph of the equation.

23. $y = -3x + 2$ **24.** $y = 2x - 3$

25. $y = 1 - x^2$ **26.** $y = x^2 - 1$

27. $y = x^2 - 3x$ **28.** $y = -x^2 - 4x$

29. $y = x^3 + 2$ **30.** $y = x^3 - 1$

31. $y = \sqrt{x - 3}$ **32.** $y = \sqrt{1 - x}$

33. $y = |x - 2|$ **34.** $y = 4 - |x|$

35. $x = y^2 - 1$ **36.** $x = y^2 - 4$

In Exercises 37–50, use a graphing utility to graph the equation. Use a standard viewing window. Approximate any *x*- or *y*-intercepts of the graph.

37. $y = x - 5$ **38.** $y = (x + 1)(x - 3)$

39. $y = 3 - \frac{1}{2}x$ **40.** $y = \frac{2}{3}x - 1$

41. $y = x^2 - 4x + 3$ **42.** $y = \frac{1}{2}(x + 4)(x - 2)$

43. $y = x(x - 2)^2$ **44.** $y = \dfrac{4}{x^2 + 1}$

45. $y = \dfrac{2x}{x - 1}$ **46.** $y = \dfrac{4}{x}$

47. $y = x\sqrt{x + 6}$ **48.** $y = (6 - x)\sqrt{x}$

49. $y = \sqrt[3]{x}$ **50.** $y = \sqrt[3]{x + 1}$

In Exercises 51–54, use a graphing utility to sketch the graph of the equation. Begin by using a standard viewing window. Then graph the equation a second time using the specified viewing window. Which viewing window is better? Explain.

51. $y = \frac{5}{2}x + 5$

Xmin = 0
Xmax = 6
Xscl = 1
Ymin = 0
Ymax = 10
Yscl = 1

52. $y = -3x + 50$

Xmin = -1
Xmax = 4
Xscl = 1
Ymin = -5
Ymax = 60
Yscl = 5

53. $y = -x^2 + 10x - 5$

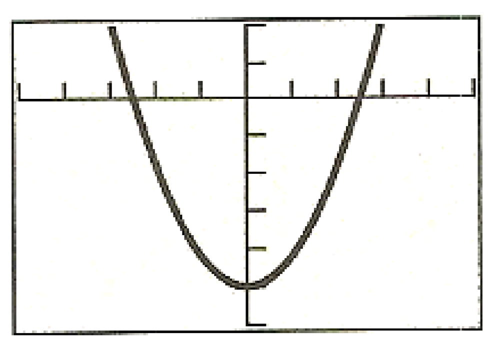
Xmin = -1
Xmax = 11
Xscl = 1
Ymin = -5
Ymax = 25
Yscl = 2

54. $y = 4(x + 5)\sqrt{4 - x}$

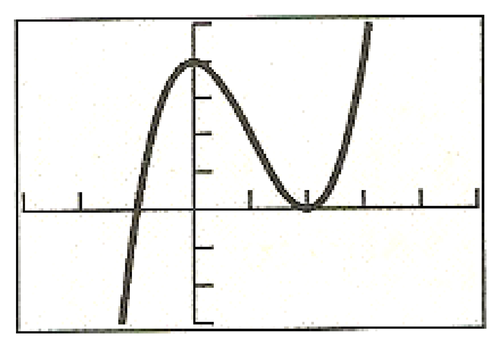
Xmin = -6
Xmax = 6
Xscl = 1
Ymin = -5
Ymax = 50
Yscl = 4

In Exercises 55–58, describe the viewing window of the graph.

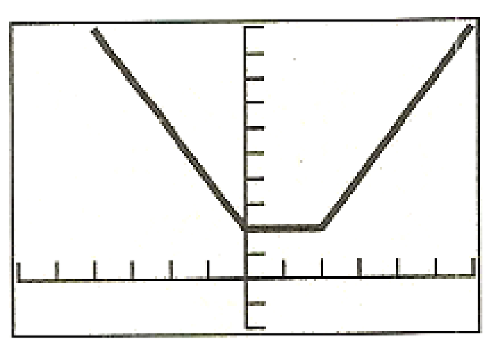
55. $y = 4x^2 - 25$



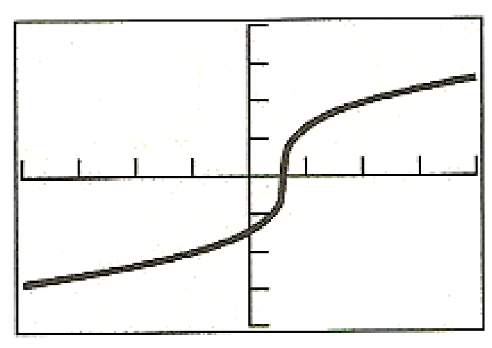
56. $y = x^3 - 3x^2 + 4$



57. $y = |x| + |x - 10|$



58. $y = 8\sqrt[3]{x - 6}$



In Exercises 59–62, solve for y and use a graphing utility to graph each of the resulting equations in the same viewing window. Adjust the viewing window so that a circle really does appear circular.

59. $x^2 + y^2 = 64$

60. $(x - 1)^2 + (y - 2)^2 = 16$

61. $x^2 + y^2 = 49$

62. $(x - 3)^2 + (y - 1)^2 = 25$

In Exercises 63–66, explain how to use a graphing utility to verify that $y_1 = y_2$. Identify the rule of algebra that is illustrated.

63. $y_1 = \frac{1}{4}(x^2 - 8)$
$y_2 = \frac{1}{4}x^2 - 2$

64. $y_1 = \frac{1}{2}x + (x + 1)$
$y_2 = \frac{3}{2}x + 1$

65. $y_1 = \frac{1}{5}[10(x^2 - 1)]$
$y_2 = 2(x^2 - 1)$

66. $y_1 = (x - 3) \cdot \frac{1}{x - 3}$
$y_2 = 1$

In Exercises 67–70, use a graphing utility to graph the equation. Move the cursor along the curve to approximate the unknown coordinate of each solution point accurate to two decimal places. (*Hint:* You may need to use the *zoom* feature of the graphing utility to obtain the required accuracy.)

67. $y = \sqrt{5 - x}$
(a) $(2, y)$
(b) $(x, 3)$

68. $y = x^3(x - 3)$
(a) $(2.25, y)$
(b) $(x, 20)$

69. $y = x^5 - 5x$
(a) $(-0.5, y)$
(b) $(x, -4)$

70. $y = |x^2 - 6x + 5|$
(a) $(2, y)$
(b) $(x, 1.5)$

71. ***Business*** A manufacturing plant purchases a new molding machine for \$225,000. The depreciated value y after t years is

$y = 225{,}000 - 20{,}000t, \qquad 0 \le t \le 8.$

(a) Use the constraints of the model to determine an appropriate viewing window.
(b) Use a graphing utility to graph the equation.
(c) Use the *value* feature or *zoom* and *trace* features of your graphing utility to determine the value of y when $t = 5.8$. Verify your answer algebraically.
(d) Use the *value* feature or *zoom* and *trace* features of your graphing utility to determine the value of y when $t = 2.35$. Verify your answer algebraically.

72. ***Consumerism*** You purchase a personal watercraft for \$8100. The depreciated value y after t years is

$y = 8100 - 929t, \qquad 0 \le t \le 6.$

(a) Use the constraints of the model to determine an appropriate viewing window.
(b) Use a graphing utility to graph the equation.
(c) Use the *value* feature or *zoom* and *trace* features of your graphing utility to determine the value of t when $y = 5545.25$. Verify your answer algebraically.
(d) Use the *value* feature or *zoom* and *trace* features of your graphing utility to determine the value of y when $t = 5.5$. Verify your answer algebraically.

73. ***Geometry*** A rectangle of length x and width w has a perimeter of 12 meters.
(a) Draw a diagram to represent the rectangle. Use the specified variables to label its sides.
(b) Show that $w = 6 - x$ is the width of the rectangle and that $A = x(6 - x)$ is its area.
(c) Use a graphing utility to graph the area equation.
(d) Use the *zoom* and *trace* features of your graphing utility to determine the value of A when $w = 4.9$ meters. Verify your answer algebraically.
(e) From the graph in part (c), estimate the dimensions of the rectangle that yield a maximum area.

74. ***Federal Debt*** The table shows the per capita U.S. federal debt for several years. (Sources: U.S. Treasury Department; U.S. Bureau of the Census)

Year	1950	1960	1970	1980
Per Capita Debt	\$1688	\$1572	\$1807	\$3981

Year	1990	1994	1997	1998
Per Capita Debt	\$12,848	\$15,750	\$20,063	\$20,513

A model for the per capita debt during this period is

$y = 0.223t^3 - 0.733t^2 - 78.255t + 1837.433$

where y represents the per capita debt and t is the time in years, with $t = 0$ corresponding to 1950.
(a) Use the *value* feature or *zoom* and *trace* features of your graphing utility to find the per capita federal debt in 1975 and 1992. Verify your answers algebraically.
(b) Use your graphing utility to determine during which year the per capita federal debt exceeded \$10,200.
(c) Use the model to estimate the per capita federal debt in 2002 and 2004.

75. ***Population Statistics*** The table gives the life expectancy of a child (at birth) in the United States for selected years from 1920 to 2000. (Source: U.S. National Center for Health Statistics)

Year	1920	1930	1940	1950
Life Expectancy	54.1	59.7	62.9	68.2

Year	1960	1970	1980	1990	2000
Life Expectancy	69.7	70.8	73.7	75.4	76.4

A model for the life expectancy during this period is

$$y = \frac{66.93 + t}{1 + 0.01t}$$

where y represents the life expectancy and t is the time in years, with $t = 0$ corresponding to 1950.

(a) What does the y-intercept of the graph of the model represent?

(b) Use your graphing utility to determine the year when the life expectancy was 73.2. Verify your answer algebraically.

(c) Determine the life expectancy in 1948 both graphically and algebraically.

(d) Use the model to estimate the life expectancy of a child born in 2005.

76. ***Finance*** The dividends declared per share of Procter & Gamble Company from 1992 to 1998 can be approximated by the model

$$y = 0.464 + 0.091t, \qquad 0 \le t \le 6$$

where y is the dividend (in dollars) and t is the time (in years), with $t = 0$ corresponding to 1992. (Source: Procter & Gamble Company)

(a) Create a table showing the dividends y for the years 1992 through 1998. From your table, determine the year during which the dividend was \$0.65.

(b) Use your graphing utility to graph the model.

(c) Verify your answer to part (a) by using the *zoom* and *trace* features of your graphing utility to determine the year when $y = 0.65$.

(d) Determine the value of y in 1997 algebraically.

(e) Use the model to estimate the value of y in 2002.

77. ***Copper Wire*** The resistance y in ohms of 1000 feet of solid copper wire at 77°F can be approximated by the mathematical model

$$y = \frac{10{,}770}{x^2} - 0.37, \qquad 5 \le x \le 100$$

where x is the diameter of the wire in mils (0.001 in.). (Source: American Wire Gage)

(a) Complete the table.

x	10	20	30	40	50	60	70	80	90	100
y										

(b) Use your table to approximate the value of x when the resistance is 4.8 ohms. Then determine the answer algebraically.

(c) Use the *value* feature or *zoom* and *trace* features of your graphing utility to determine the resistance when $x = 85.5$.

(d) What can you conclude in general about the relationship between the diameter of the copper wire and the resistance?

Synthesis

True or False? **In Exercises 78 and 79, determine whether the statement is true or false. Justify your answer.**

78. A parabola can have only one x-intercept.

79. The graph of a linear equation can have either no x-intercepts or only one x-intercept.

80. ***Think About It*** Find a and b if the x-intercepts of the graph of $y = (x - a)(x - b)$ are $(-2, 0)$ and $(5, 0)$.

81. ***Writing*** Explain how to find an appropriate viewing window for the graph of an equation.

P.3 Lines in the Plane

The Slope of a Line

In this section, you will study lines and their equations. The **slope** of a nonvertical line represents the number of units a line rises or falls vertically for each unit of horizontal change from left to right. For instance, consider the two points (x_1, y_1) and (x_2, y_2) on the line shown in Figure P.26. As you move from left to right along this line, a change of $(y_2 - y_1)$ units in the vertical direction corresponds to a change of $(x_2 - x_1)$ units in the horizontal direction. That is,

$$y_2 - y_1 = \text{the change in } y$$

and

$$x_2 - x_1 = \text{the change in } x.$$

The slope of the line is given by the ratio of these two changes.

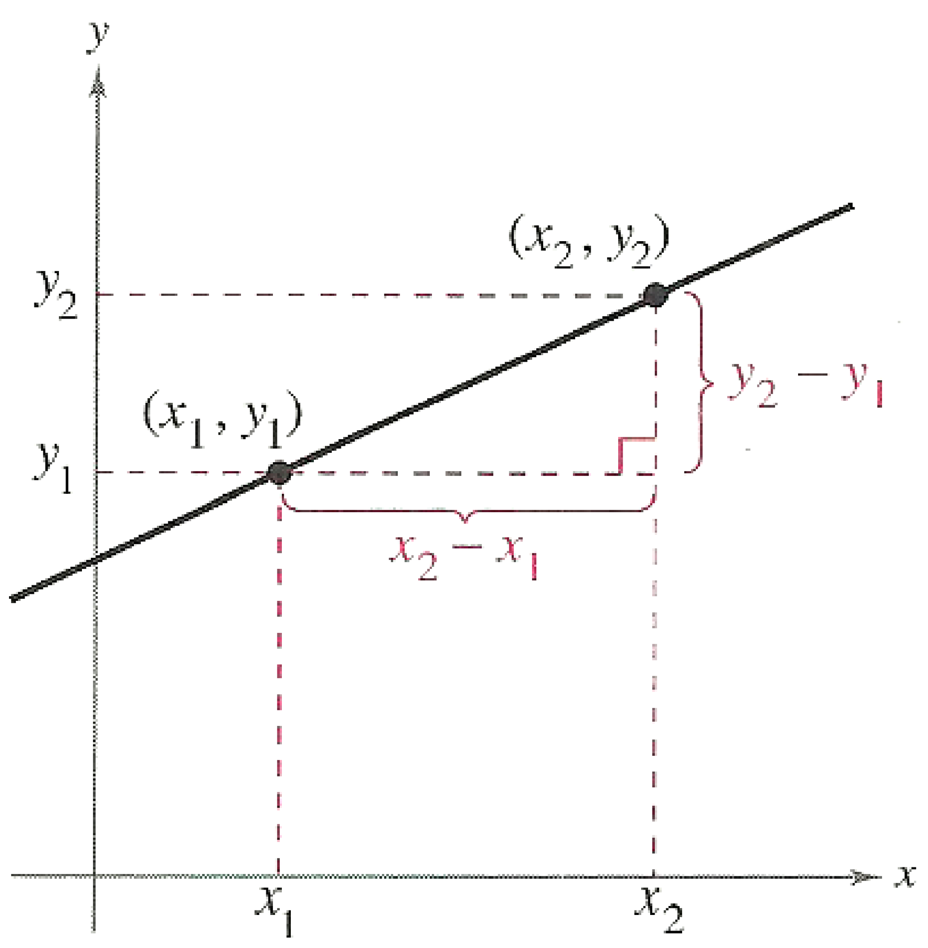


Figure P.26

Definition of the Slope of a Line

The **slope** m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\text{change in } y}{\text{change in } x}$$

where $x_1 \neq x_2$.

When this formula is used, the *order of subtraction* is important. Given two points on a line, you are free to label either one of them as (x_1, y_1) and the other as (x_2, y_2). However, once this has been done, you must form the numerator and denominator using the same order of subtraction.

$$\underbrace{m = \frac{y_2 - y_1}{x_2 - x_1}}_{\text{Correct}} \qquad \underbrace{m = \frac{y_1 - y_2}{x_1 - x_2}}_{\text{Correct}} \qquad \underbrace{\cancel{m = \frac{y_2 - y_1}{x_1 - x_2}}}_{\text{Incorrect}}$$

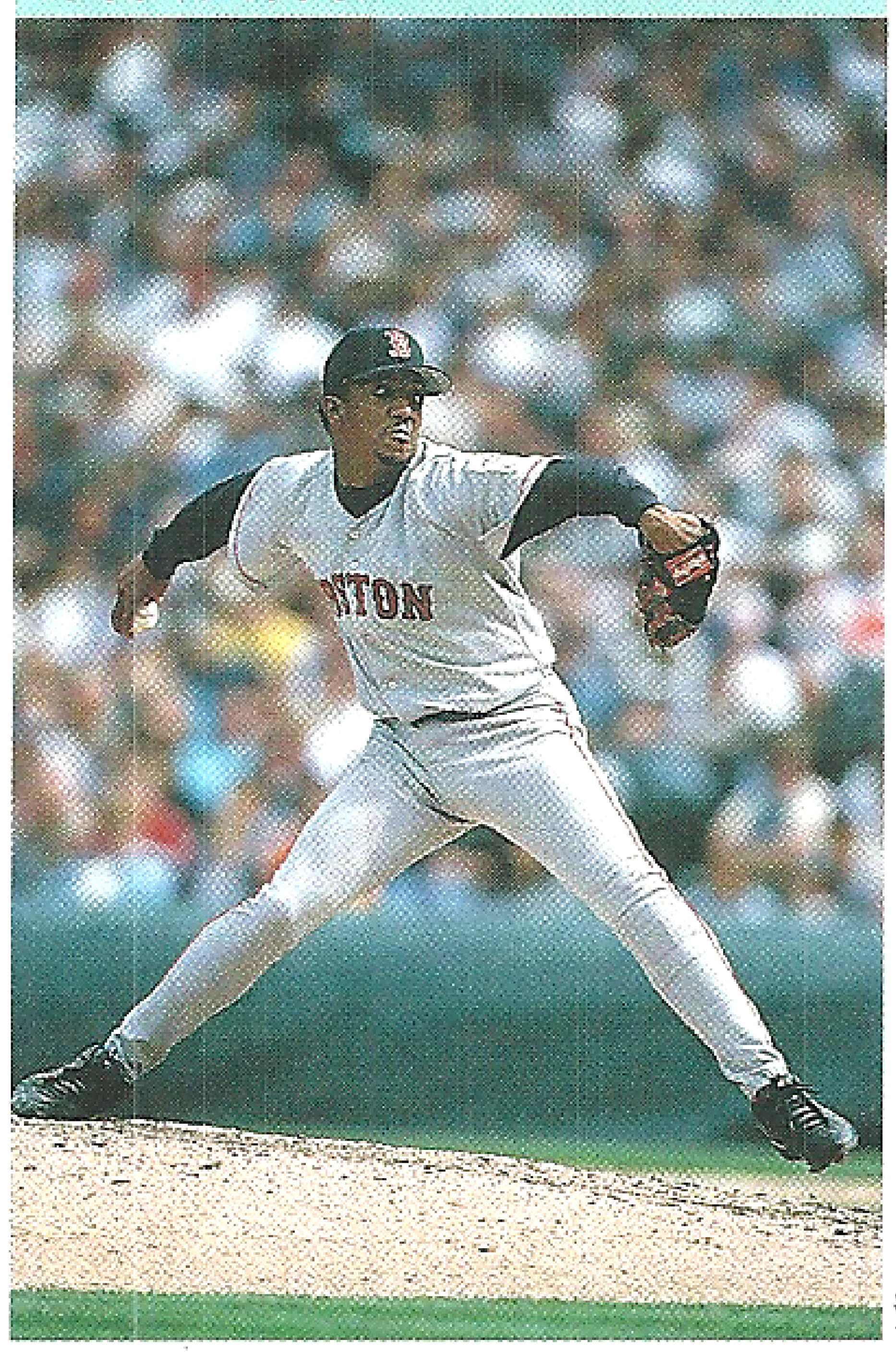
Throughout this text, the term *line* always means a *straight* line.

What You Should Learn:

- How to find the slopes of lines
- How to write linear equations given points on lines and their slopes
- How to use slope-intercept forms of linear equations to sketch graphs of lines
- How to use slope to identify parallel and perpendicular lines

Why You Should Learn It:

Linear equations can be used to model and solve real-life problems. For instance, Exercise 95 on page 37 shows how to use a linear equation to model the average annual salaries of Major League Baseball players from 1988 to 1998.



Allsport

EXAMPLE 1 Finding the Slope of a Line

Find the slope of the line passing through each pair of points.

a. $(-2, 0)$ and $(3, 1)$ **b.** $(-1, 2)$ and $(2, 2)$ **c.** $(0, 4)$ and $(1, -1)$

Solution

$$\textbf{a. } m = \frac{\overbrace{y_2 - y_1}^{\text{Difference in } y\text{-values}}}{\underbrace{x_2 - x_1}_{\text{Difference in } x\text{-values}}} = \frac{1 - 0}{3 - (-2)} = \frac{1}{3 + 2} = \frac{1}{5}$$

$$\textbf{b. } m = \frac{2 - 2}{2 - (-1)} = \frac{0}{3} = 0$$

$$\textbf{c. } m = \frac{-1 - 4}{1 - 0} = \frac{-5}{1} = -5$$

The graphs of the three lines are shown in Figure P.27. Note that the square setting gives the correct "steepness" of the lines.

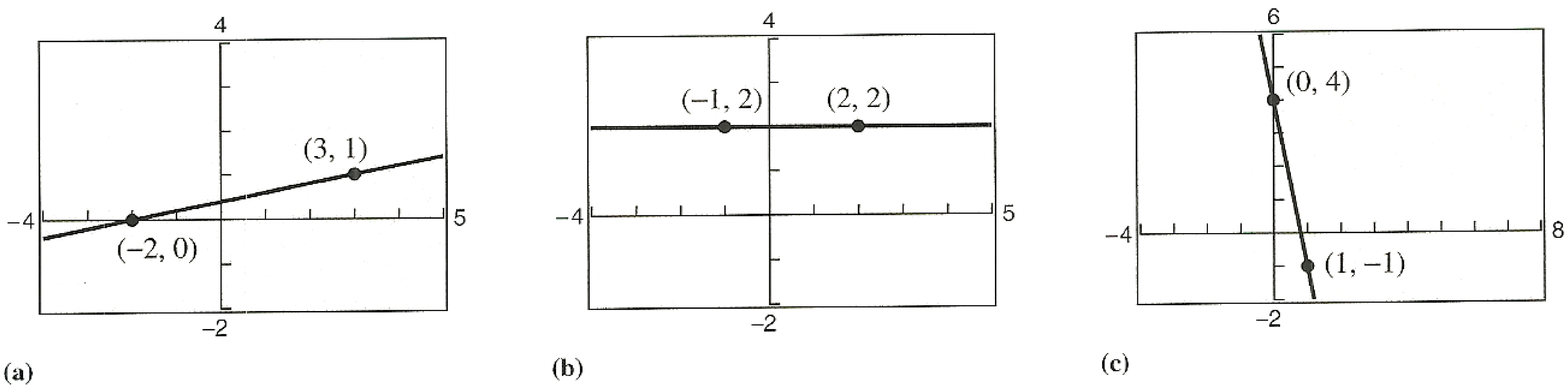


Figure P.27

Exploration

Use a graphing utility to compare the slopes of the lines $y = 0.5x$, $y = x$, $y = 2x$, and $y = 4x$. What do you observe about these lines? Compare the slopes of the lines $y = -0.5x$, $y = -x$, $y = -2x$, and $y = -4x$. What do you observe about these lines? (*Hint:* Use a square setting to guarantee a true geometric perspective.)

The definition of slope does not apply to vertical lines. For instance, consider the points $(3, 4)$ and $(3, 1)$ on the vertical line shown in Figure P.28. Applying the formula for slope, you obtain

$$m = \frac{4 - 1}{3 - 3} = \cancel{\frac{3}{0}}. \qquad \text{Undefined}$$

Because division by zero is undefined, the slope of a vertical line is undefined.

From the slopes of the lines shown in Figures P.27 and P.28, you can make the following generalizations about the slope of a line.

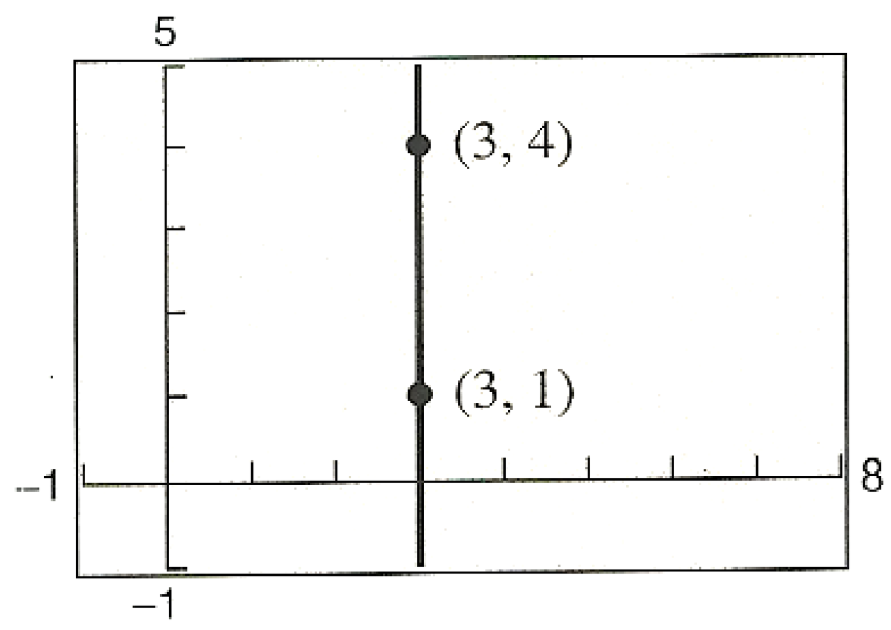


Figure P.28

The Slope of a Line

1. A line with positive slope $(m > 0)$ *rises* from left to right.
2. A line with negative slope $(m < 0)$ *falls* from left to right.
3. A line with zero slope $(m = 0)$ is *horizontal*.
4. A line with undefined slope is *vertical*.

The Point-Slope Form of the Equation of a Line

If you know the slope of a line *and* you also know the coordinates of one point on the line, you can find an equation for the line. For instance, in Figure P.29, let (x_1, y_1) be a given point on the line whose slope is m. If (x, y) is any *other* point on the line, it follows that

$$\frac{y - y_1}{x - x_1} = m.$$

This equation in the variables x and y can be rewritten in the **point-slope form** of the equation of a line.

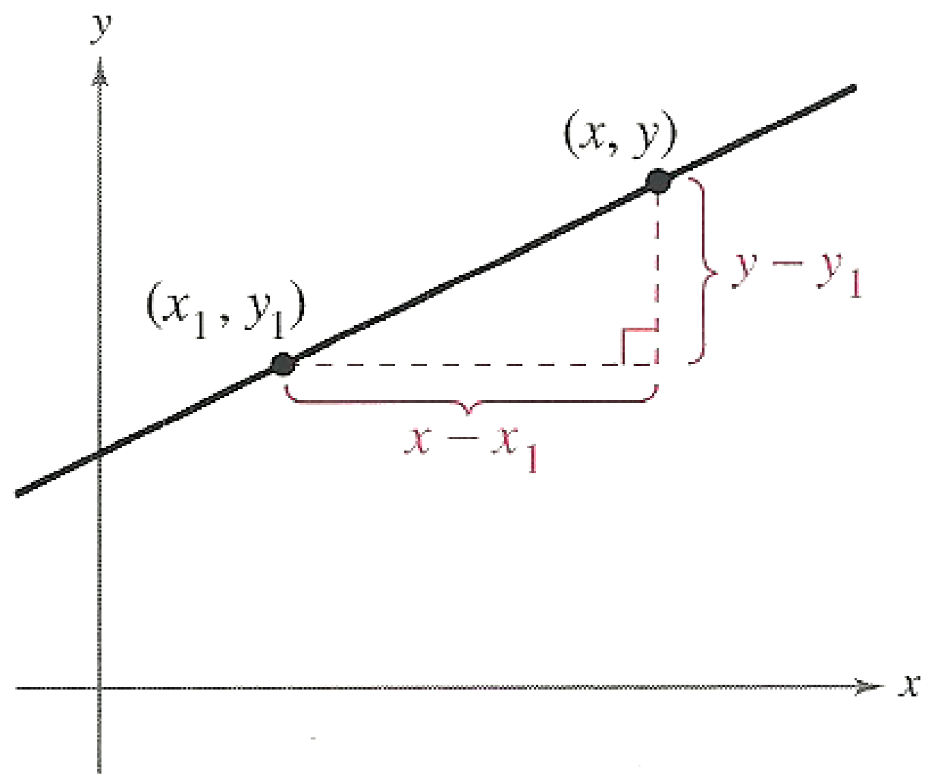


Figure P.29

Point-Slope Form of the Equation of a Line

The **point-slope form** of the equation of the line that passes through the point (x_1, y_1) and has a slope of m is

$$y - y_1 = m(x - x_1).$$

EXAMPLE 2 The Point-Slope Form of the Equation of a Line

Find an equation of the line that passes through the point $(1, -2)$ and has a slope of 3.

Solution

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$
$$y - (-2) = 3(x - 1) \qquad \text{Substitute for } y_1, m, \text{ and } x_1.$$
$$y + 2 = 3x - 3$$
$$y = 3x - 5$$

This line is shown in Figure P.30.

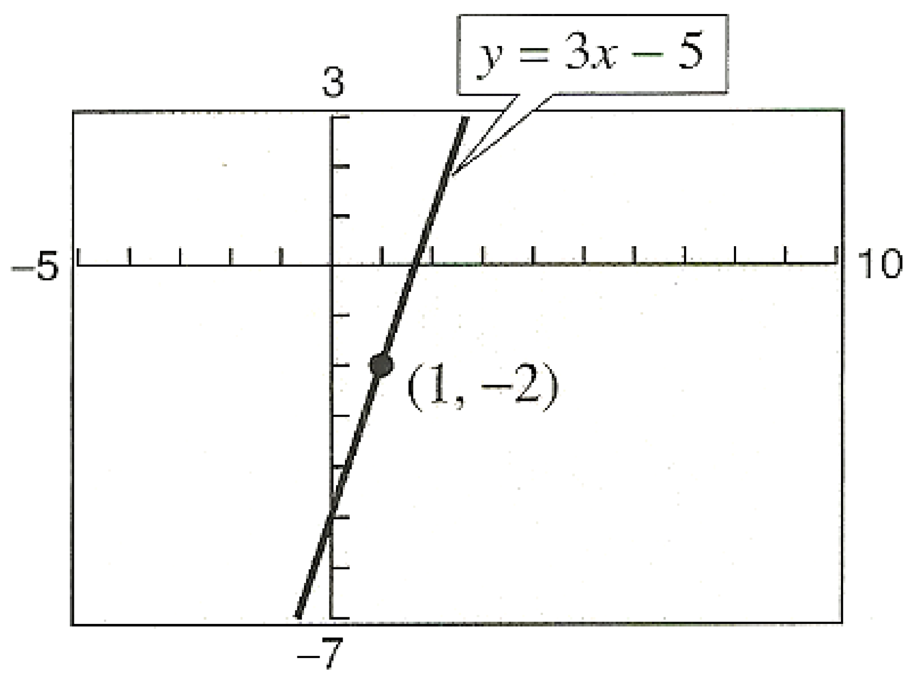


Figure P.30

The point-slope form can be used to find an equation of a nonvertical line passing through two points (x_1, y_1) and (x_2, y_2). First, use the formula for the slope of the line passing through two points.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Then, once you know the slope, use the point-slope form to obtain the equation

$$y - y_1 = m(x - x_1)$$
$$= \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

This is sometimes called the **two-point form** of the equation of a line.

EXAMPLE 3 A Linear Model for Sales Prediction

During 1997, Barnes & Noble's net sales were \$2.8 billion, and in 1998 net sales were \$3.0 billion. (Source: Barnes & Noble, Inc.)

a. Write a linear equation giving the net sales y in terms of the year x.

b. Use the equation to estimate the net sales during 2000.

Solution

a. Let $x = 7$ represent 1997. In Figure P.31, let $(7, 2.8)$ and $(8, 3.0)$ be two points on the line representing the net sales. The slope of the line passing through these two points is

$$m = \frac{3.0 - 2.8}{8 - 7} = 0.2. \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

By the point-slope form, the equation of the line is as follows.

$$y - y_1 = m(x - x_1) \qquad \text{Point-slope form}$$

$$y - 2.8 = 0.2(x - 7) \qquad \text{Substitute for } y_1, m, \text{ and } x_1.$$

$$y = 0.2x - 1.4 + 2.8$$

$$y = 0.2x + 1.4 \qquad \text{Simplify.}$$

b. Using the equation from part (a), estimate the 2000 net sales ($x = 10$) to be

$$y = 0.2(10) + 1.4$$

$$= 2 + 1.4$$

$$= \$3.4 \text{ billion.}$$

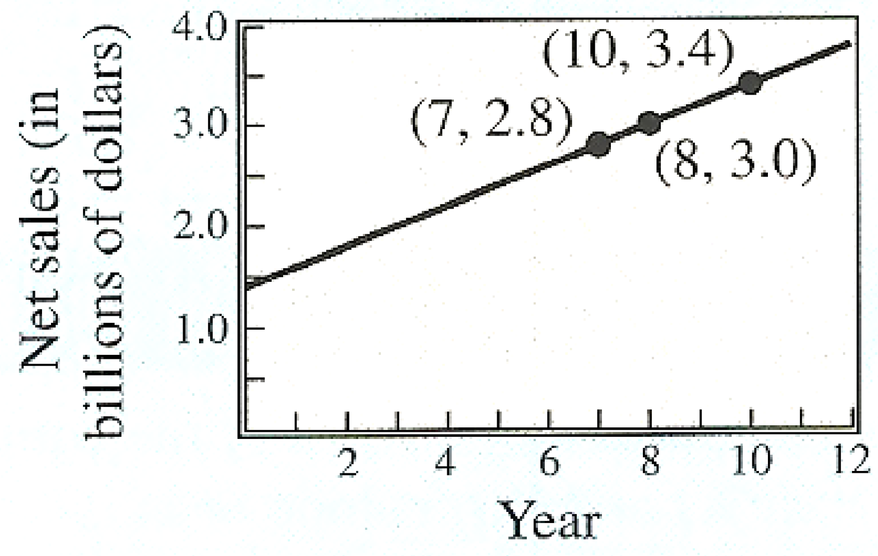
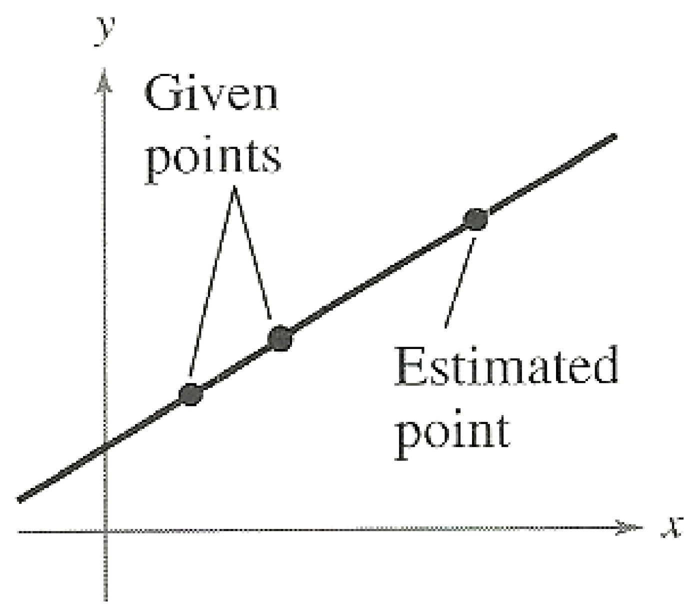
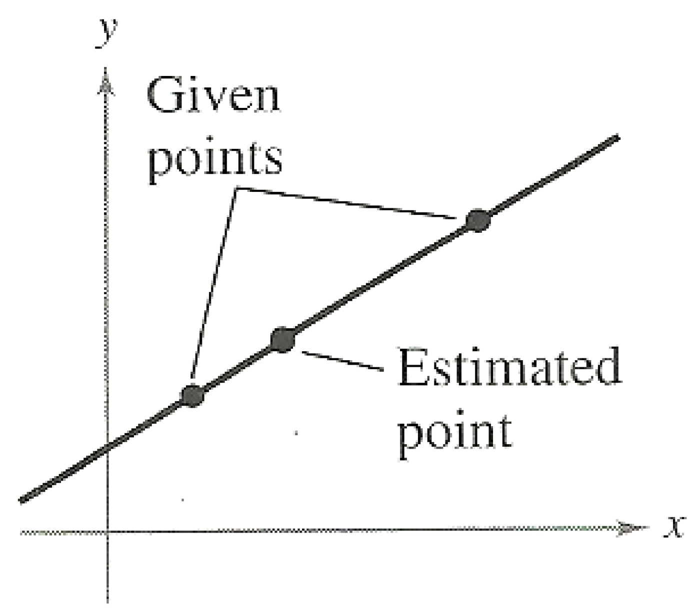


Figure P.31

The approximation method illustrated in Example 3 is **linear extrapolation.** Note in Figure P.32 that for linear extrapolation, the estimated point lies outside of the given points. When the estimated point lies *between* two given points, the procedure is called **linear interpolation.**



Linear Extrapolation



Linear Interpolation

Figure P.32

Library of Functions

In Section 1.1, you will be introduced to the precise meaning of the term *function*. The simplest type of function is a linear function and has the form

$$y = mx + b.$$

As its name implies, the graph of a linear function is a line that has a slope of m and a y-intercept at $(0, b)$.

Consult the Library of Functions Summary inside the front cover for a description of the linear function.

Sketching Graphs of Lines

Many problems in coordinate geometry can be classified as follows.

1. Given a graph (or parts of it), find its equation.
2. Given an equation, sketch its graph.

For lines, the first problem is solved easily by using the point-slope form. This formula, however, is not particularly useful for solving the second type of problem. The form that is better suited to graphing linear equations is the **slope-intercept form,** $y = mx + b$, of the equation of a line.

Slope-Intercept Form of the Equation of a Line

The graph of the equation

$$y = mx + b$$

is a line whose slope is m and whose y-intercept is $(0, b)$.

A computer simulation of this concept appears in the *Interactive* CD-ROM and *Internet* versions of this text.

EXAMPLE 4 Using the Slope-Intercept Form

Determine the slope and y-intercept of each linear equation. Then describe its graph.

a. $x + y = 2$ **b.** $y = 2$

Algebraic Solution

a. Begin by writing the equation in slope-intercept form.

$x + y = 2$ Write original equation.

$y = 2 - x$ Subtract x from each side.

$y = -x + 2$ Slope-intercept form

From the slope-intercept form of the equation, the slope is -1 and the y-intercept is $(0, 2)$. Because the slope is negative, you know that the graph of the equation is a line that falls one unit for every unit it moves to the right.

b. By writing the equation $y = 2$ in slope-intercept form

$$y = (0)x + 2$$

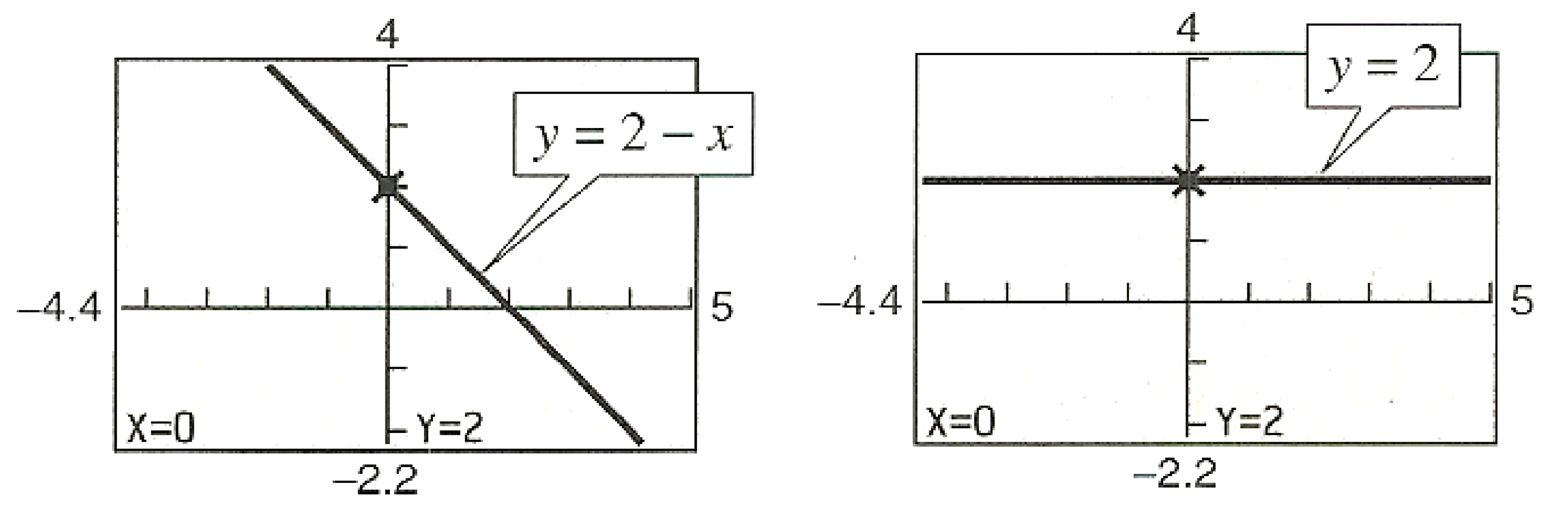
you can see that the slope is 0 and the y-intercept is $(0, 2)$. A zero slope implies that the line is horizontal.

Graphical Solution

a. Solve the equation for y to obtain $y = 2 - x$. Enter this equation in your graphing utility. Use a decimal viewing window to graph the equation as shown in Figure P.33(a).

To find the y-intercept, use the *value* or *trace* feature. When $x = 0$, $y = 2$. So, the y-intercept is $(0, 2)$. To find the slope, continue to use the *trace* feature. Move the cursor along the line until $x = 1$. At this point, $y = 1$. So the graph falls 1 unit for every unit it moves to the right, and the slope is -1.

b. Enter the equation $y = 2$ in your graphing utility and graph the equation as shown in Figure P.33(b). Use the *trace* feature to verify the y-intercept $(0, 2)$ and to see that the value of y is the same for all values of x. So, the slope of the horizontal line is 0.



(a) **(b)**

Figure P.33

From the slope-intercept form of the equation of a line, you can see that a horizontal line ($m = 0$) has an equation of the form $y = b$. This is consistent with the fact that each point on a horizontal line through $(0, b)$ has a y-coordinate of b.

Similarly, each point on a vertical line through $(a, 0)$ has an x-coordinate of a. So, a vertical line has an equation of the form $x = a$. This equation cannot be written in the slope-intercept form, because the slope of a vertical line is undefined. However, *every* line has an equation that can be written in the **general form**

$$Ax + By + C = 0 \qquad \text{General form of the equation of a line}$$

where A and B are not *both* zero.

Exploration

Graph the lines $y = 2x + 1$, $y = \frac{1}{2}x + 1$, and $y = -2x + 1$ in the same viewing window. What do you observe?

Graph the lines $y = 2x + 1$, $y = 2x$, and $y = 2x - 1$ in the same viewing window. What do

Summary of Equations of Lines

1. **General form:** $Ax + By + C = 0$
2. **Vertical line:** $x = a$
3. **Horizontal line:** $y = b$
4. **Slope-intercept form:** $y = mx + b$
5. **Point-slope form:** $y - y_1 = m(x - x_1)$

EXAMPLE 5 Different Viewing Windows

The graphs of the two lines

$$y = -x - 1 \quad \text{and} \quad y = -10x - 1$$

are shown in Figure P.34. Even though the slopes of these lines are quite different (-1 and -10, respectively), the graphs seem misleadingly similar because the viewing windows are different.

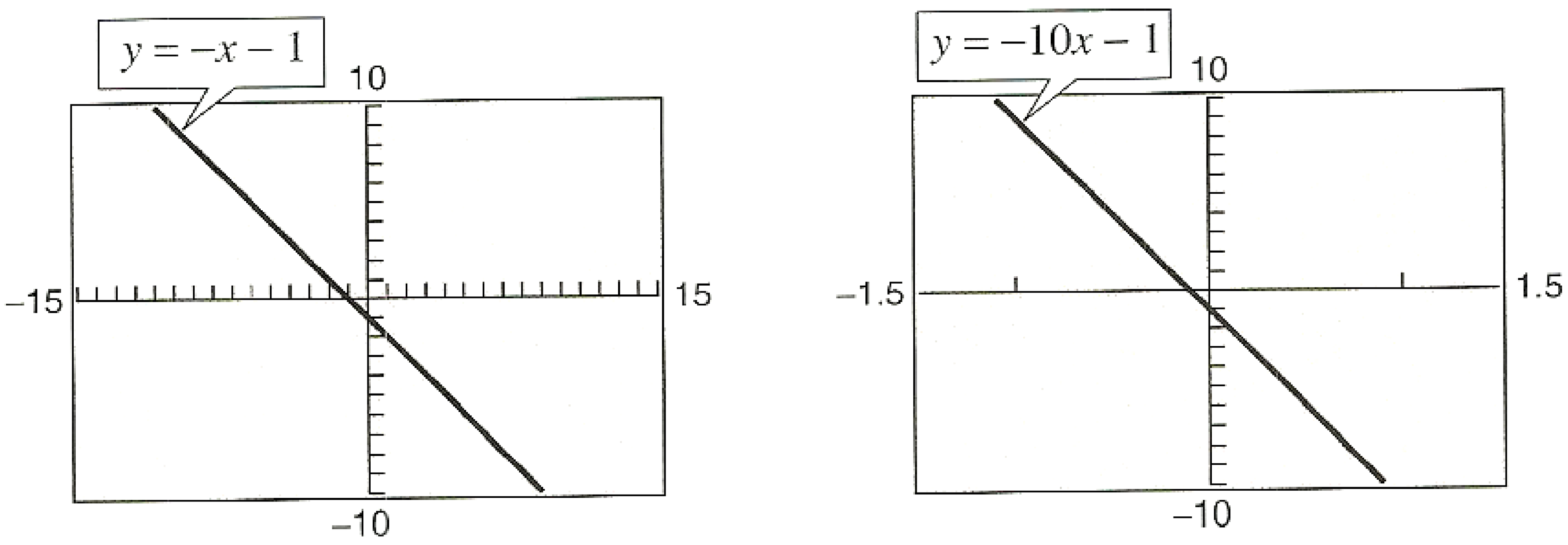


Figure P.34

When a graphing utility is used to sketch a straight line, it is important to realize that the graph of the line may not visually appear to have the slope indicated by its equation. This occurs because of the viewing window used for the graph. For instance, Figure P.35 shows graphs of $y = 2x + 1$ produced on a graphing utility using three different viewing windows.

Notice that the slopes in Figure P.35(a) and (b) do not visually appear to be equal to 2. However, if you use the *square* viewing window, as in Figure P.35(c), the slope visually appears to be 2. In general, two graphs of the same equation can appear to be quite different depending on the viewing window selected.

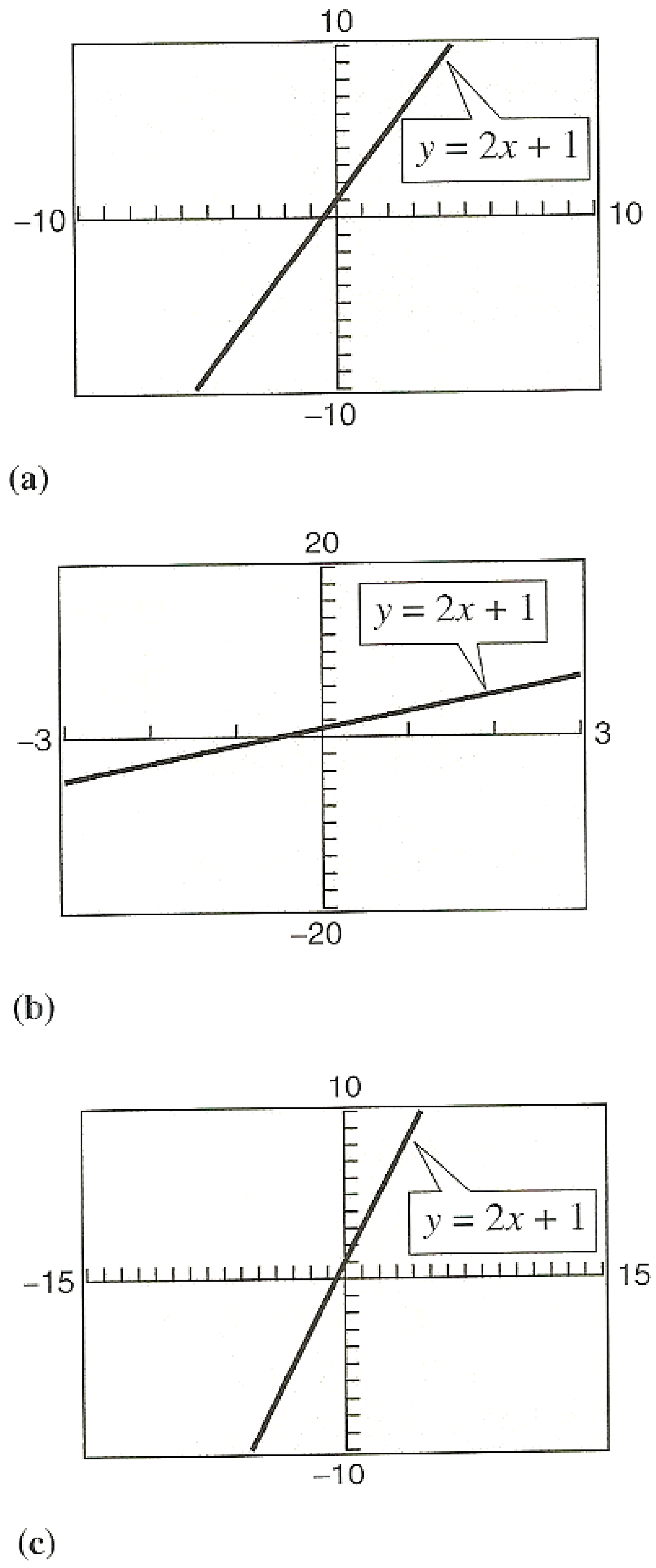


Figure P.35

Parallel and Perpendicular Lines

The slope of a line is a convenient tool for determining whether two lines are parallel or perpendicular.

Parallel Lines

Two distinct nonvertical lines are **parallel** if and only if their slopes are equal.

EXAMPLE 6 Equations of Parallel Lines

Find the slope-intercept form of the equation of the line that passes through the point $(2, -1)$ and is parallel to the line $2x - 3y = 5$.

Solution

Begin by finding the slope of the given line.

$$2x - 3y = 5 \qquad \text{Write original equation.}$$
$$-2x + 3y = -5 \qquad \text{Multiply by } -1.$$
$$3y = 2x - 5 \qquad \text{Add } 2x \text{ to each side.}$$
$$y = \frac{2}{3}x - \frac{5}{3} \qquad \text{Write in slope-intercept form.}$$

Therefore, the given line has a slope of $m = \frac{2}{3}$. Because any line parallel to the given line must also have a slope of $\frac{2}{3}$, the required line through $(2, -1)$ has the following equation.

$$y - (-1) = \frac{2}{3}(x - 2) \qquad \text{Substitute for } m, x_1, \text{ and } y_1 \text{ in point-slope form.}$$
$$y + 1 = \frac{2}{3}x - \frac{4}{3} \qquad \text{Simplify.}$$
$$y = \frac{2}{3}x - \frac{4}{3} - 1 \qquad \text{Subtract 1 from each side.}$$
$$y = \frac{2}{3}x - \frac{7}{3} \qquad \text{Write in slope-intercept form.}$$

Notice the similarity between the slope-intercept form of the original equation and the slope-intercept form of the parallel equation. The graphs of both equations are shown in Figure P.36.

STUDY TIP

Be careful when you graph equations such as $y = \frac{2}{3}x - \frac{7}{3}$ on your graphing utility. A common mistake is to type it in as

Y1 = 2/3X − 7/3,

which may not be interpreted as the original equation by your graphing utility. You should use one of the following formulas.

Y1 = 2X/3 − 7/3

Y1 = (2/3)X − 7/3

Do you see why?

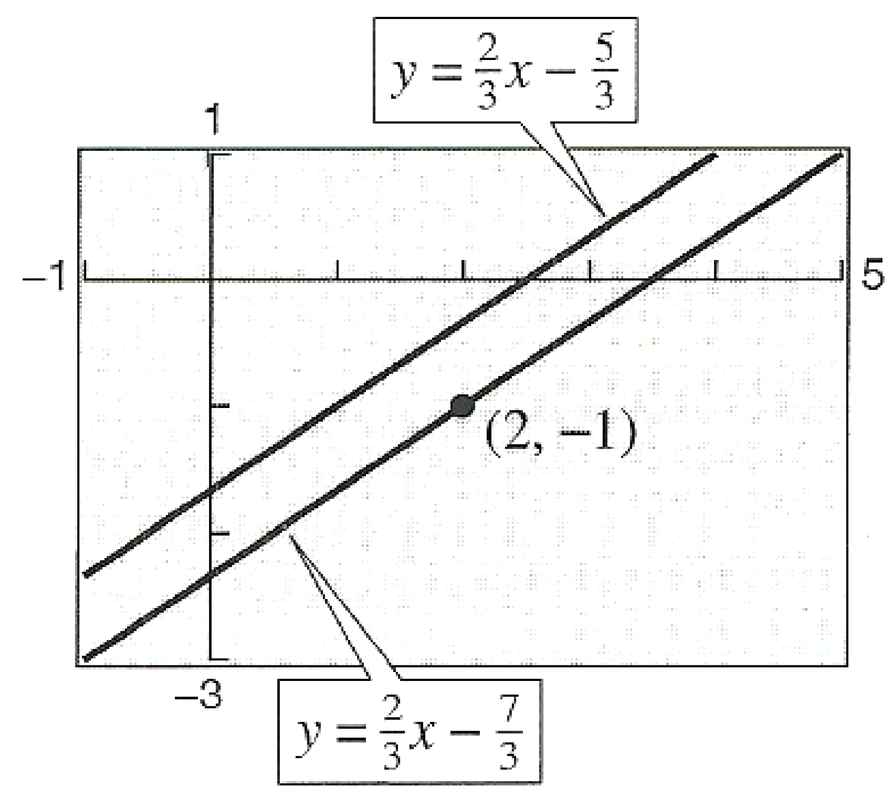


Figure P.36

Perpendicular Lines

Two nonvertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other. That is,

$$m_1 = -\frac{1}{m_2}.$$

EXAMPLE 7 Equations of Perpendicular Lines

Find an equation of the line that passes through the point $(2, -1)$ and is perpendicular to the line $2x - 3y = 5$.

Solution

By writing the given line in the form $y = \frac{2}{3}x - \frac{5}{3}$, you can see that the line has a slope of $\frac{2}{3}$. So, any line that is perpendicular to this line must have a slope of $-\frac{3}{2}$ (because $-\frac{3}{2}$ is the negative reciprocal of $\frac{2}{3}$). Therefore, the required line through the point $(2, -1)$ has the following equation.

$$y - (-1) = -\tfrac{3}{2}(x - 2) \qquad \text{Substitute for } m, x_1, \text{ and } y_1 \text{ in point-slope form.}$$

$$y + 1 = -\tfrac{3}{2}x + 3 \qquad \text{Simplify.}$$

$$y = -\tfrac{3}{2}x + 3 - 1 \qquad \text{Subtract 1 from each side.}$$

$$y = -\tfrac{3}{2}x + 2 \qquad \text{Slope-intercept form}$$

The graphs of both equations are shown in Figure P.37.

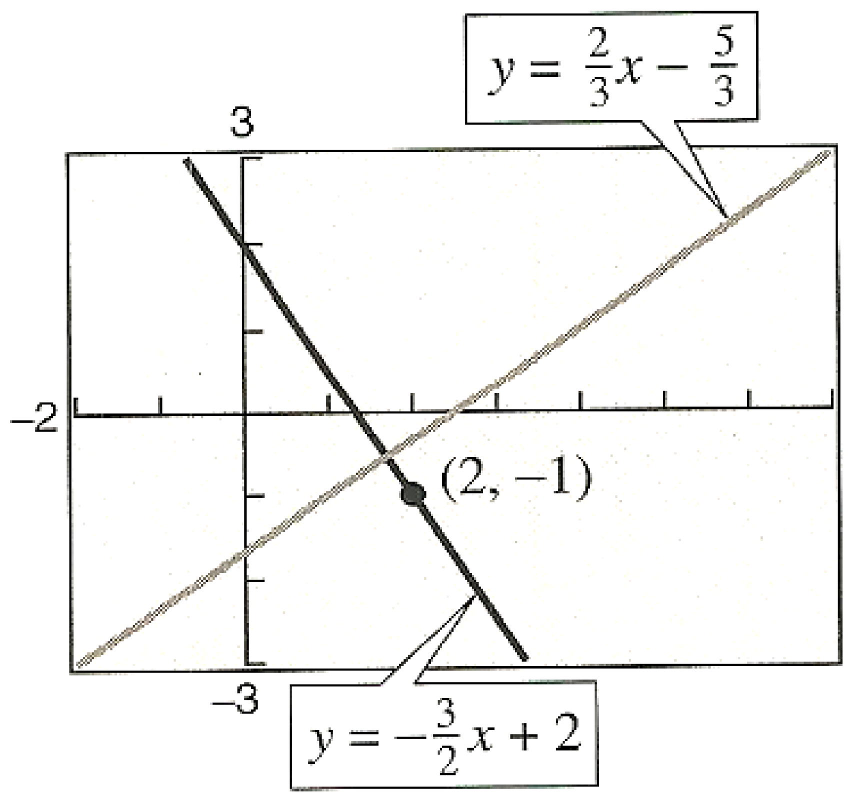


Figure P.37

EXAMPLE 8 Graphs of Perpendicular Lines

Use a graphing utility to graph the lines

$y = x + 1$

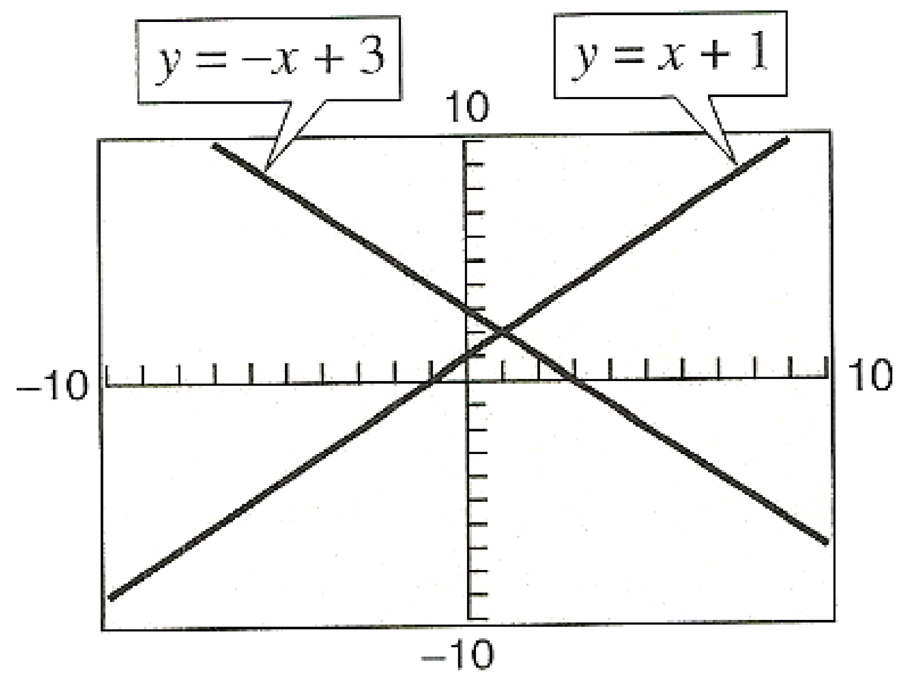
and

$y = -x + 3$.

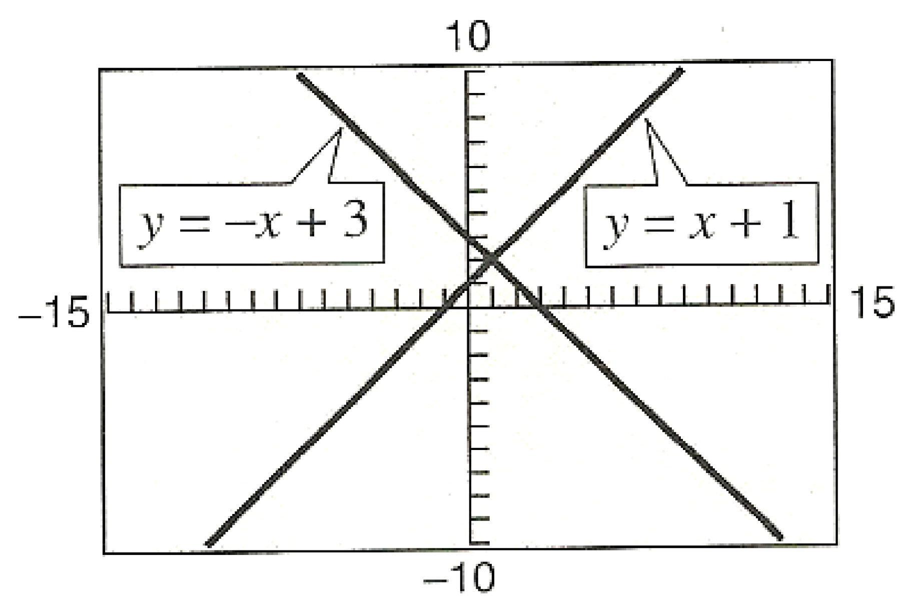
Display *both* graphs in the same viewing window. The lines are supposed to be perpendicular (they have slopes of $m_1 = 1$ and $m_2 = -1$). Do they appear to be perpendicular on the display?

Solution

If the viewing window is nonsquare, as in Figure P.38(a), the two lines will not appear perpendicular. If, however, the viewing window is square, as in Figure P.38(b), the lines will appear perpendicular.



(a)



(b)

Figure P.38

Writing About Math *An Application of Slope*

In 1990, a college had an enrollment of 5500 students. By 2000, the enrollment had increased to 7000 students.

a. What was the average annual change in enrollment from 1990 to 2000?

b. Use the average annual change in enrollment to estimate the enrollments in 1993, 1997, and 1999.

c. Write the equation of the line that represents the data in part (b). What is its slope? Interpret the slope in the context of the problem.

d. Write a short paragraph discussing the concepts of *slope* and *average rate of change.*

P.3 Exercises

In Exercises 1 and 2, identify the line that has the specified slope.

1. (a) $m = \frac{2}{3}$ (b) m is undefined. (c) $m = -2$

2. (a) $m = 0$ (b) $m = -\frac{3}{4}$ (c) $m = 1$

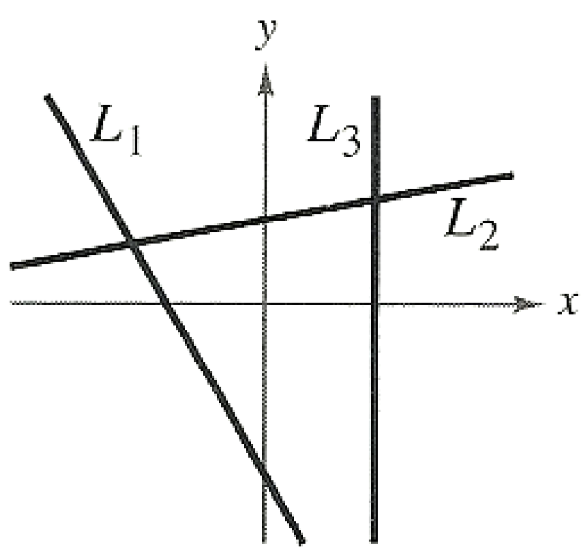


FIGURE FOR 1

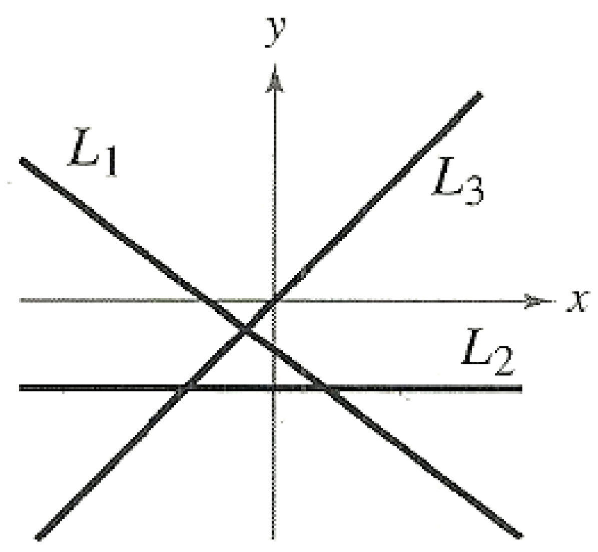
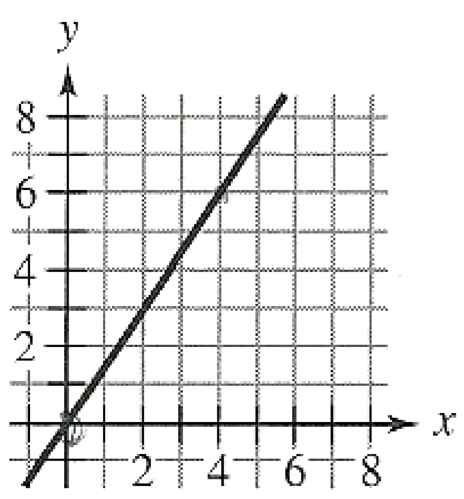


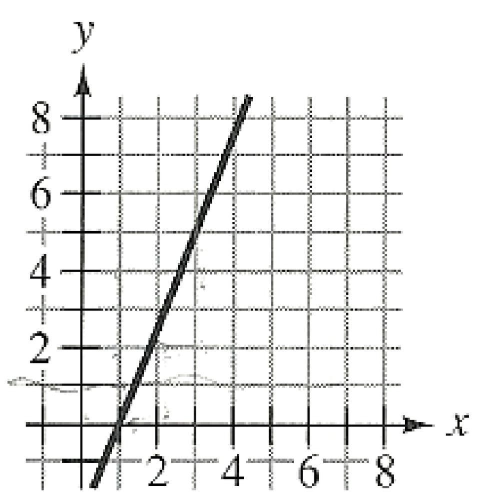
FIGURE FOR 2

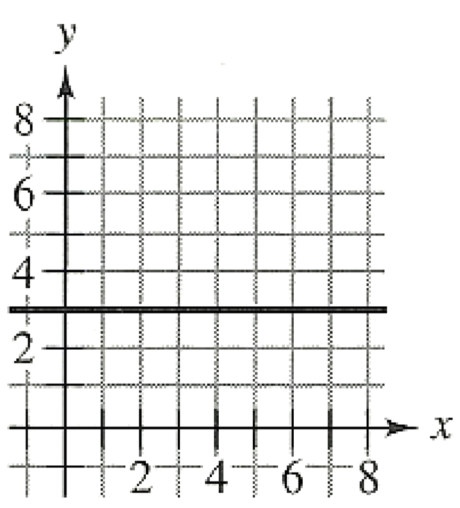
In Exercises 3 and 4, sketch the line through the point with each indicated slope on the same set of coordinate axes.

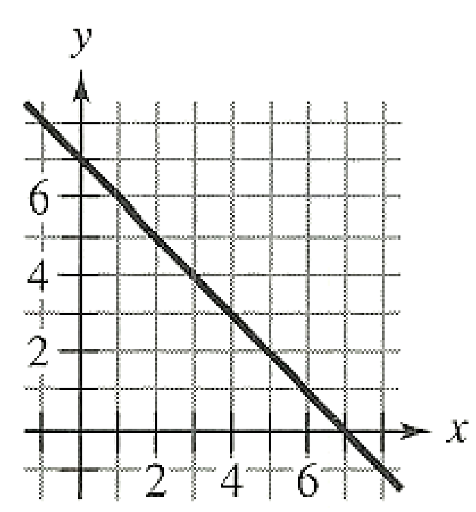
Point	*Slopes*			
3. (2, 3)	(a) 0	(b) 1	(c) 2	(d) -3
4. $(-4, 1)$	(a) 3	(b) -3	(c) $\frac{1}{2}$	(d) Undefined

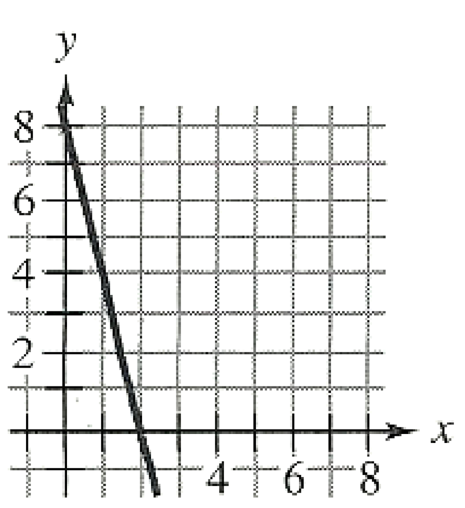
In Exercises 5–10, estimate the slope of the line.

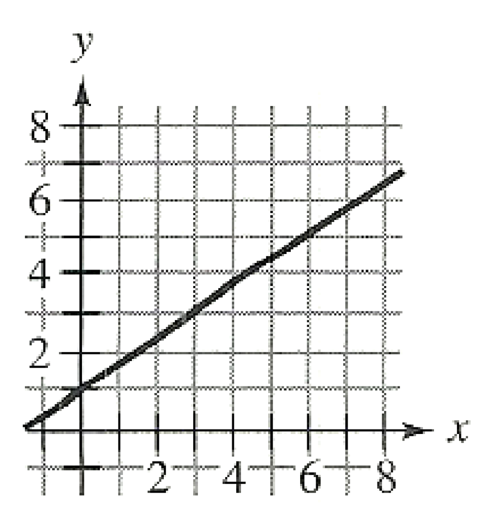
5.


6.


7.


8.


9.


10.


In Exercises 11–14, plot the points and find the slope of the line passing through the points. Verify the slope using the *draw* feature on your graphing utility to graph the line segment connecting the two points. (Use a square setting.)

11. $(0, -10), (-4, 0)$ **12.** $(2, 4), (4, -4)$

13. $(-6, -1), (-6, 4)$ **14.** $(-3, -2), (1, 6)$

In Exercises 15–20, you are given the slope of the line and a point on the line. Find three additional points through which the line passes. (There are many correct answers.)

Point	*Slope*
15. (2, 1)	$m = 0$
16. $(-4, 1)$	m is undefined.
17. $(-5, 4)$	$m = 2$
18. $(0, -9)$	$m = -2$
19. $(7, -2)$	$m = \frac{1}{2}$
20. $(-1, -6)$	$m = -\frac{1}{2}$

In Exercises 21–24, determine whether the lines L_1 and L_2 passing through the pairs of points are parallel, perpendicular, or neither. Use a graphing utility to graph the line segments connecting the pairs of points on the respective lines. (Use a square setting.)

21. L_1: $(0, -1), (5, 9)$
L_2: $(0, 3), (4, 1)$

22. L_1: $(-2, -1), (1, 5)$
L_2: $(1, 3), (5, -5)$

23. L_1: $(3, 6), (-6, 0)$
L_2: $(0, -1), \left(5, \frac{7}{3}\right)$

24. L_1: $(4, 8), (-4, 2)$
L_2: $(3, -5), \left(-1, \frac{1}{3}\right)$

In Exercises 25–32, (a) find the slope and y-intercept (if possible) of the equation of the line algebraically, (b) sketch the line by hand, and (c) use a graphing utility to verify your answers to parts (a) and (b).

25. $5x - y + 3 = 0$ **26.** $2x + 3y - 9 = 0$

27. $5x - 2 = 0$ **28.** $3x + 7 = 0$

29. $3y + 5 = 0$ **30.** $-11 - 8y = 0$

31. $7x + 6y - 30 = 0$ **32.** $x - y - 10 = 0$

In Exercises 33–42, find the general form of the equation of the line that passes through the given point and has the indicated slope. Sketch the line by hand. Use a graphing utility to verify your sketch.

	Point	*Slope*
33.	$(0, -2)$	$m = 3$
34.	$(0, 10)$	$m = -1$
35.	$(-3, 6)$	$m = -2$
36.	$(0, 0)$	$m = 4$
37.	$(4, 0)$	$m = -\frac{1}{3}$
38.	$(-2, -5)$	$m = \frac{3}{4}$
39.	$(6, -1)$	m is undefined.
40.	$(-10, 4)$	$m = 0$
41.	$\left(-\frac{1}{2}, \frac{3}{2}\right)$	$m = -3$
42.	$(2.3, -8.5)$	$m = -\frac{5}{2}$

In Exercises 43–52, find the general form of the equation of the line that passes through the points. Use a graphing utility to sketch the line.

43. $(5, -1), (-5, 5)$ **44.** $(4, 3), (-4, -4)$

45. $(-8, 1), (-8, 7)$ **46.** $(-1, 4), (6, 4)$

47. $\left(2, \frac{1}{2}\right), \left(\frac{1}{2}, \frac{5}{4}\right)$ **48.** $(1, 1), \left(6, -\frac{2}{3}\right)$

49. $\left(-\frac{1}{10}, -\frac{3}{5}\right), \left(\frac{9}{10}, -\frac{9}{5}\right)$ **50.** $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

51. $(1, 0.6), (-2, -0.6)$ **52.** $(-8, 0.6), (2, -2.4)$

Exploration **In Exercises 53 and 54, use the values of a and b and a graphing utility to graph the equation of the line**

$$\frac{x}{a} + \frac{y}{b} = 1, \qquad a \neq 0, b \neq 0.$$

Use the graphs to make a conjecture about what a and b represent. Verify your conjecture.

53. $a = 5, \quad b = -3$ **54.** $a = -6, \quad b = 2$

In Exercises 55–58, use the results of Exercises 53 and 54 to write an equation of the line that passes through the points.

55. x-intercept: $(2, 0)$
y-intercept: $(0, 3)$

56. x-intercept: $(-5, 0)$
y-intercept: $(0, -4)$

57. x-intercept: $\left(-\frac{1}{6}, 0\right)$
y-intercept: $\left(0, -\frac{2}{3}\right)$

58. x-intercept: $\left(\frac{3}{4}, 0\right)$
y-intercept: $\left(0, \frac{4}{5}\right)$

In Exercises 59 and 60, use a graphing utility to graph the equation using each of the suggested viewing windows. Describe the difference between the two graphs.

59. $y = 0.5x - 3$

Xmin = -5	Xmin = -2
Xmax = 10	Xmax = 10
Xscl = 1	Xscl = 1
Ymin = -1	Ymin = -4
Ymax = 10	Ymax = 1
Yscl = 1	Yscl = 1

60. $y = -8x + 5$

Xmin = -5	Xmin = -5
Xmax = 5	Xmax = 10
Xscl = 1	Xscl = 1
Ymin = -10	Ymin = -80
Ymax = 10	Ymax = 80
Yscl = 1	Yscl = 20

Graphical Analysis **In Exercises 61–64, use a graphing utility to graph the three equations in the same viewing window. Adjust the viewing window so that the slope appears visually correct. Identify any relationships that exist among the lines. Use the slope of the lines to verify your results.**

61. (a) $y = 2x$ (b) $y = -2x$ (c) $y = \frac{1}{2}x$

62. (a) $y = \frac{2}{3}x$ (b) $y = -\frac{3}{2}x$ (c) $y = \frac{2}{3}x + 2$

63. (a) $y = -\frac{1}{2}x$ (b) $y = -\frac{1}{2}x + 3$ (c) $y = 2x - 4$

64. (a) $y = x - 8$ (b) $y = x + 1$ (c) $y = -x + 3$

In Exercises 65–70, write equations of the lines through the given point (a) parallel to the given line and (b) perpendicular to the given line.

	Point	*Line*
65.	$(2, 1)$	$4x - 2y = 3$
66.	$(-3, 2)$	$x + y = 7$
67.	$\left(-\frac{2}{3}, \frac{7}{8}\right)$	$3x + 4y = 7$
68.	$\left(\frac{7}{8}, \frac{3}{4}\right)$	$5x + 3y = 0$
69.	$(2.5, 6.8)$	$x - y = 4$
70.	$(-3.9, -1.4)$	$6x + 2y = 9$

In Exercises 71 and 72, find a relationship between x and y such that (x, y) is equidistant from the two points.

71. $(4, -1), (-2, 3)$ **72.** $(3, -2), (-7, 1)$

73. ***Business*** The slopes are the slopes of lines representing annual sales y in terms of time x in years. Use each slope to interpret any change in annual sales for a 1-year increase in time.

(a) The line has a slope of $m = 135$.

(b) The line has a slope of $m = 0$.

(c) The line has a slope of $m = -40$.

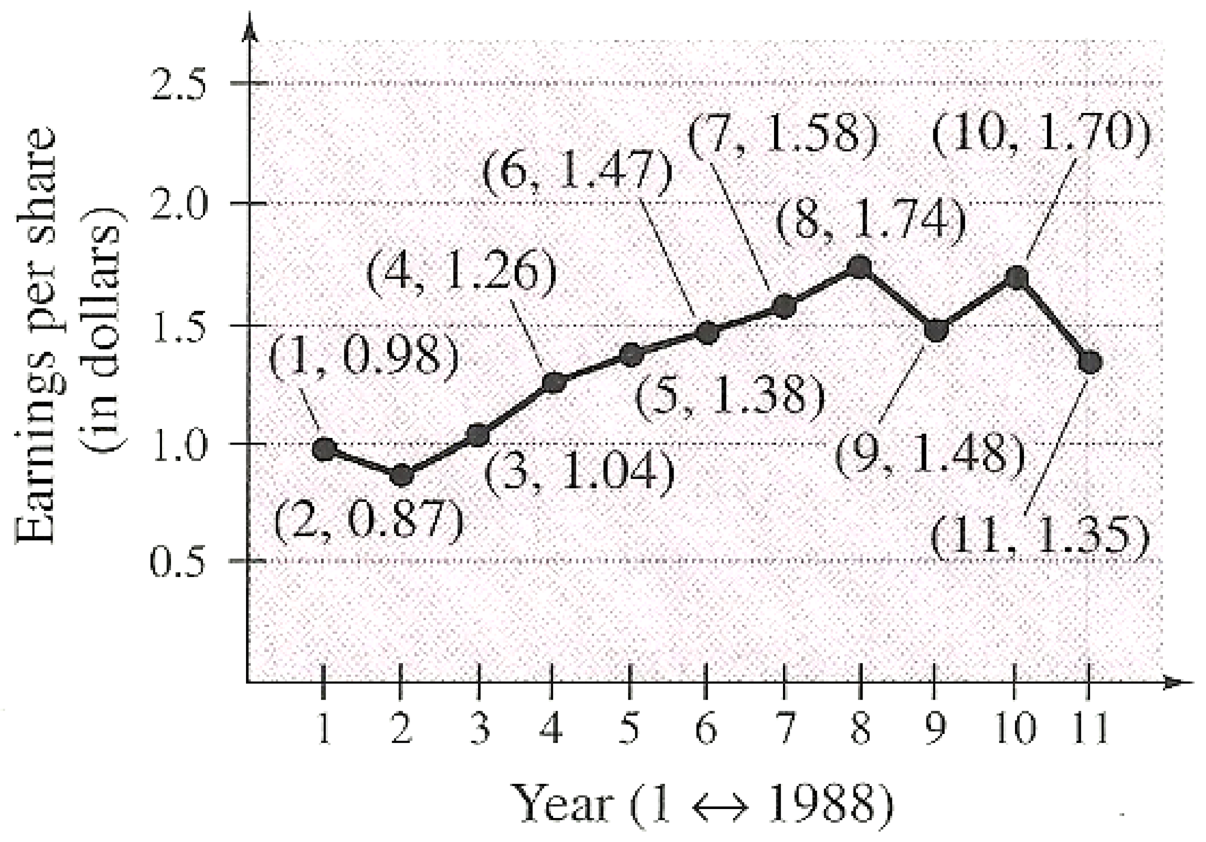
74. ***Business*** The slopes are the slopes of lines representing daily revenues y in terms of time x in days. Use each slope to interpret any change in daily revenues for a 1-day increase in time.

(a) The line has a slope of $m = 400$.

(b) The line has a slope of $m = 100$.

(c) The line has a slope of $m = 0$.

75. ***Business*** The graph shows the earnings per share of stock for the Kellogg Company for the years 1988 through 1998. (Source: Kellogg Company)



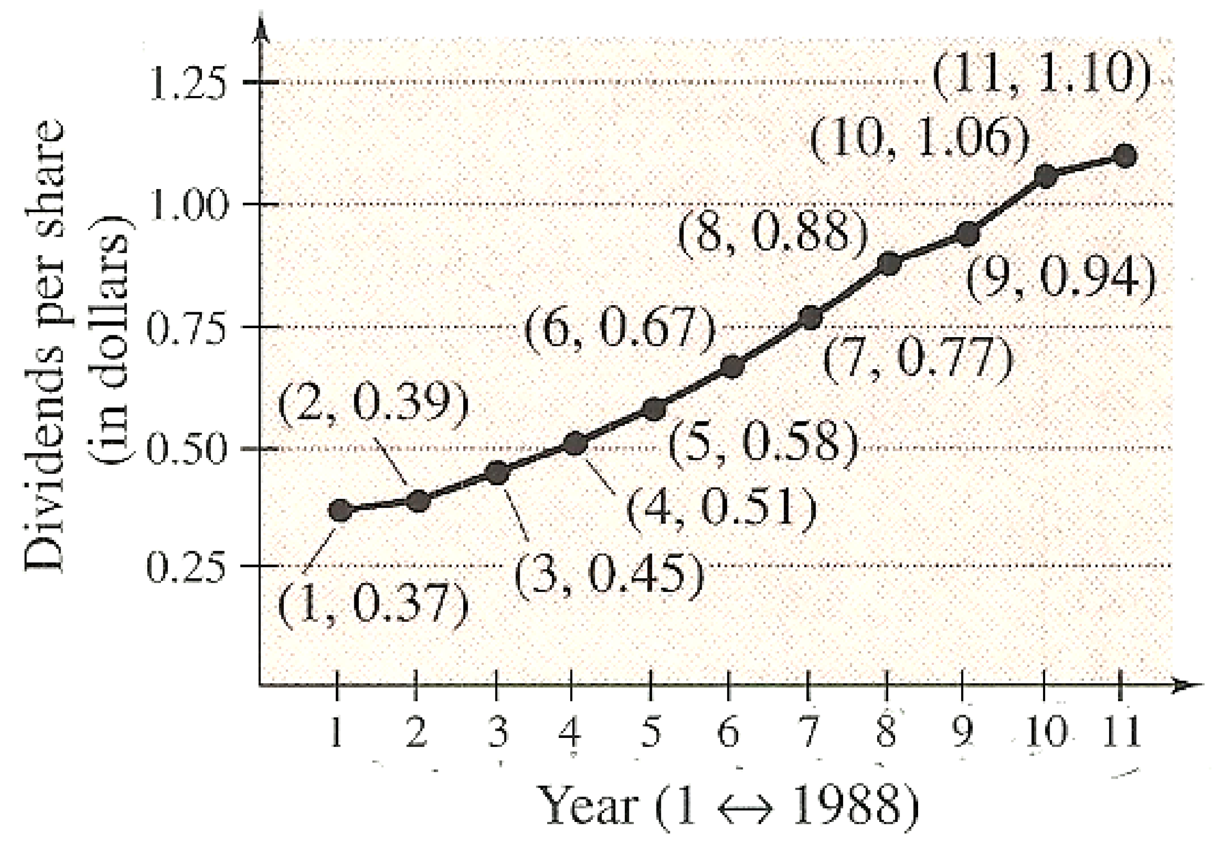
(a) Use the slopes to determine the year(s) when the earnings per share showed the greatest increase and decrease.

(b) Find the equation of the line between the years 1988 and 1998.

(c) Interpret the meaning of the slope in the equation from part (b) in the context of the problem.

(d) Use the equation from part (b) to estimate the earnings per share of stock for the year 2001. Do you think this is an accurate estimation? Explain.

76. ***Business*** The graph shows the dividends declared per share of stock for the Colgate-Palmolive Company for the years 1988 through 1998. (Source: Colgate-Palmolive Company)



(a) Use the slopes to determine the years when the dividends declared per share showed the greatest increase and the smallest increase.

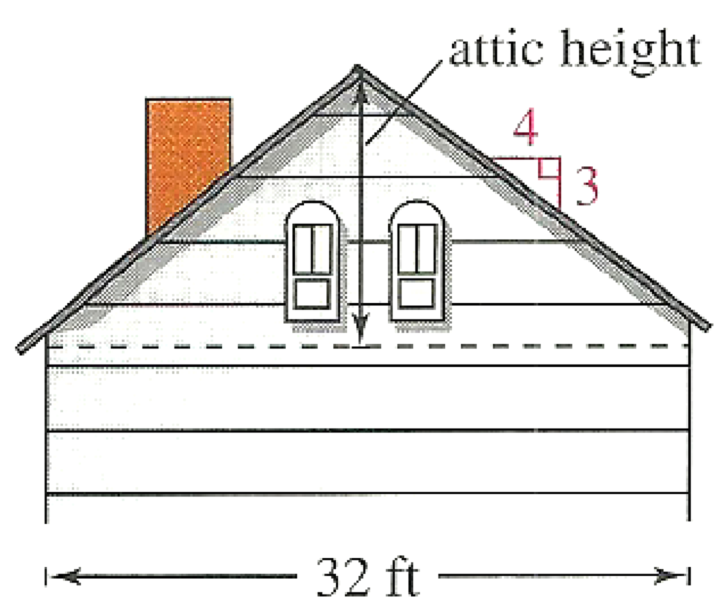
(b) Find the equation of the line between the years 1988 and 1998.

(c) Interpret the meaning of the slope in the equation from part (b) in the context of the problem.

(d) Use the equation from part (b) to estimate the dividends declared per share for the year 2001. Do you think this is an accurate estimation? Explain.

77. ***Driving*** When driving down a mountain road, you notice warning signs indicating that it is a "12% grade." This means that the slope of the road is $-\frac{12}{100}$. Approximate the amount of horizontal change in your position if you note from elevation markers that you have descended 2000 feet vertically.

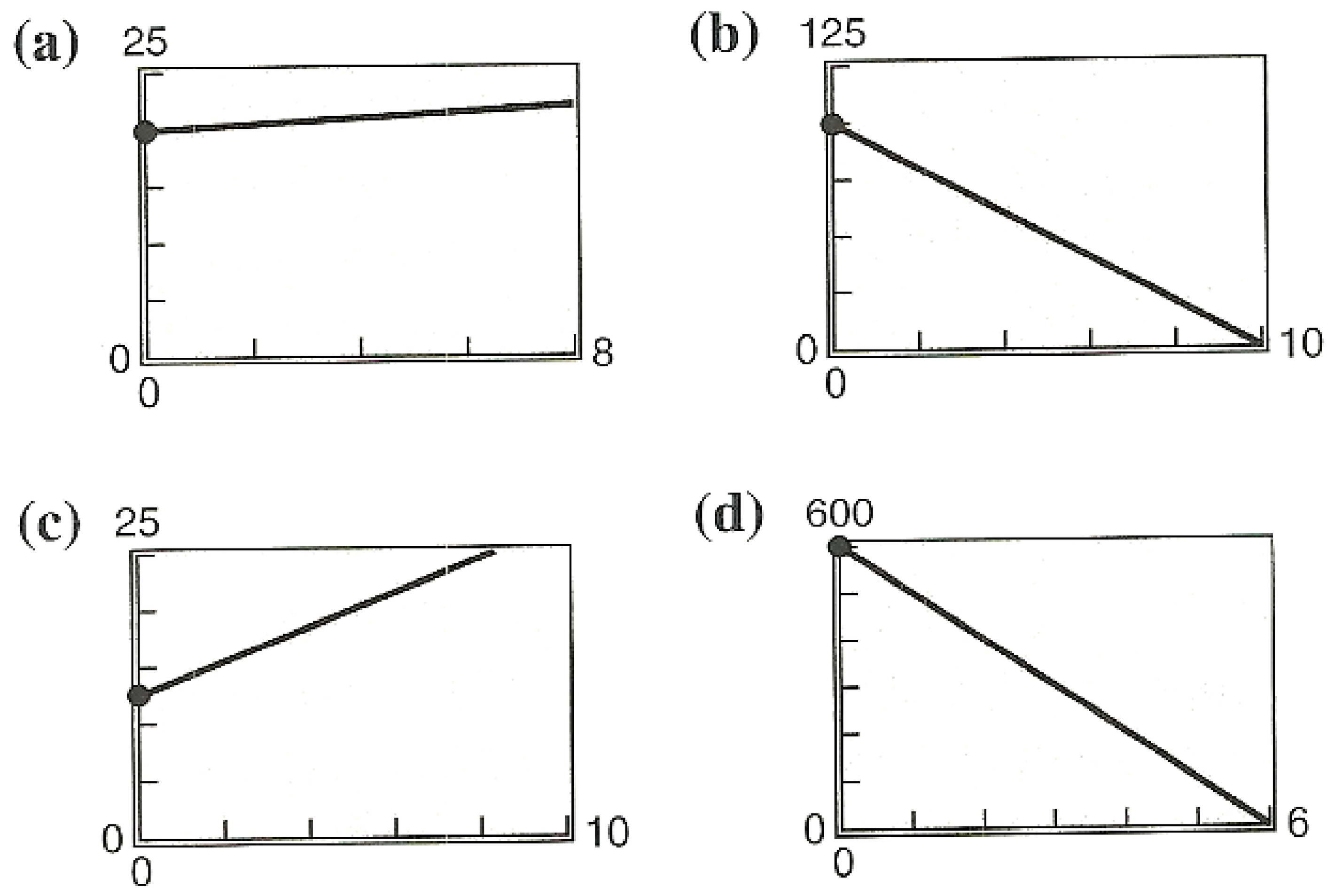
78. ***Attic Height*** The "rise to run" ratio of the roof of a house determines the steepness of the roof. Suppose the rise to run ratio of a roof is 3 to 4. Determine the maximum height in the attic of the house if the house is 32 feet wide.



Rate of Change **In Exercises 79–82, you are given the dollar value of a product in 2001 and the rate at which the value of the product is expected to change during the next 5 years. Use this information to write a linear equation that gives the dollar value V of the item in terms of the year t. (Let $t = 1$ represent 2001.)**

	2001 Value	*Rate*
79.	\$2540	\$125 increase per year
80.	\$156	\$4.50 increase per year
81.	\$20,400	\$2000 decrease per year
82.	\$245,000	\$5600 decrease per year

Graphical Interpretation **In Exercises 83–86, match the description with its graph. Also determine the slope and how it is interpreted in the situation. [The graphs are labeled (a), (b), (c), and (d).]**



83. A person is paying \$10 per week to a friend to repay a \$100 loan.

84. An employee is paid \$12.50 per hour plus \$1.50 for each unit produced per hour.

85. A sales representative receives \$20 per day for food plus \$0.25 for each mile traveled.

86. A word processor that was purchased for \$600 depreciates \$100 per year.

87. ***Temperature*** Find the equation of the line that shows the relationship between the temperature in degrees Celsius C and degrees Fahrenheit F. Remember that water freezes at 0°C (32°F) and boils at 100°C (212°F).

88. ***Temperature*** Use the result of Exercise 87 to complete the table.

C		$-10°$	$10°$			$177°$
F	$0°$			$68°$	$90°$	

89. ***Finance*** Your salary was \$28,500 in 1998 and \$32,900 in 2000. If your salary follows a linear growth pattern, what will your salary be in 2003?

90. ***College Enrollment*** A small college had 2546 students in 1998 and 2702 students in 2000. If the enrollment follows a linear growth pattern, how many students will the college have in 2004?

91. ***Business*** A small business purchases a fax machine for \$875. After 5 years, the fax machine will be outdated and have no value.

(a) Write a linear equation giving the value V of the fax machine during the 5 years it will be used.

(b) Use a graphing utility to graph the linear equation representing the depreciation of the fax machine, and use the *value* or *trace* feature to complete the table.

t	0	1	2	3	4	5
V						

(c) Verify your answers in part (b) algebraically by using the equation you found in part (a).

92. ***Business*** A small business purchases a computer network system for \$25,000. After 10 years, the system will have to be replaced. Its value at that time is expected to be \$2000.

(a) Write a linear equation giving the value V of the system during the 10 years it will be used.

(b) Use a graphing utility to graph the linear equation representing the depreciation of the system, and use the *value* or *trace* feature to complete the table.

t	0	1	2	3	4	5	6	7	8	9	10
V											

(c) Verify your answers in part (b) algebraically by using the equation you found in part (a).

93. ***Business*** A contractor purchases a bulldozer for \$36,500. The bulldozer requires an average expenditure of \$5.25 per hour for fuel and maintenance, and the operator is paid \$11.50 per hour.

(a) Write a linear equation giving the total cost C of operating the bulldozer for t hours. (Include the purchase cost of the bulldozer.)

(b) Assuming that customers are charged \$27 per hour of bulldozer use, write an equation for the revenue R derived from t hours of use.

(c) Use the formula for profit ($P = R - C$) to write an equation for the profit derived from t hours of use.

(d) Use the result of part (c) to find the break-even point (the number of hours the bulldozer must be used to yield a profit of 0 dollars).

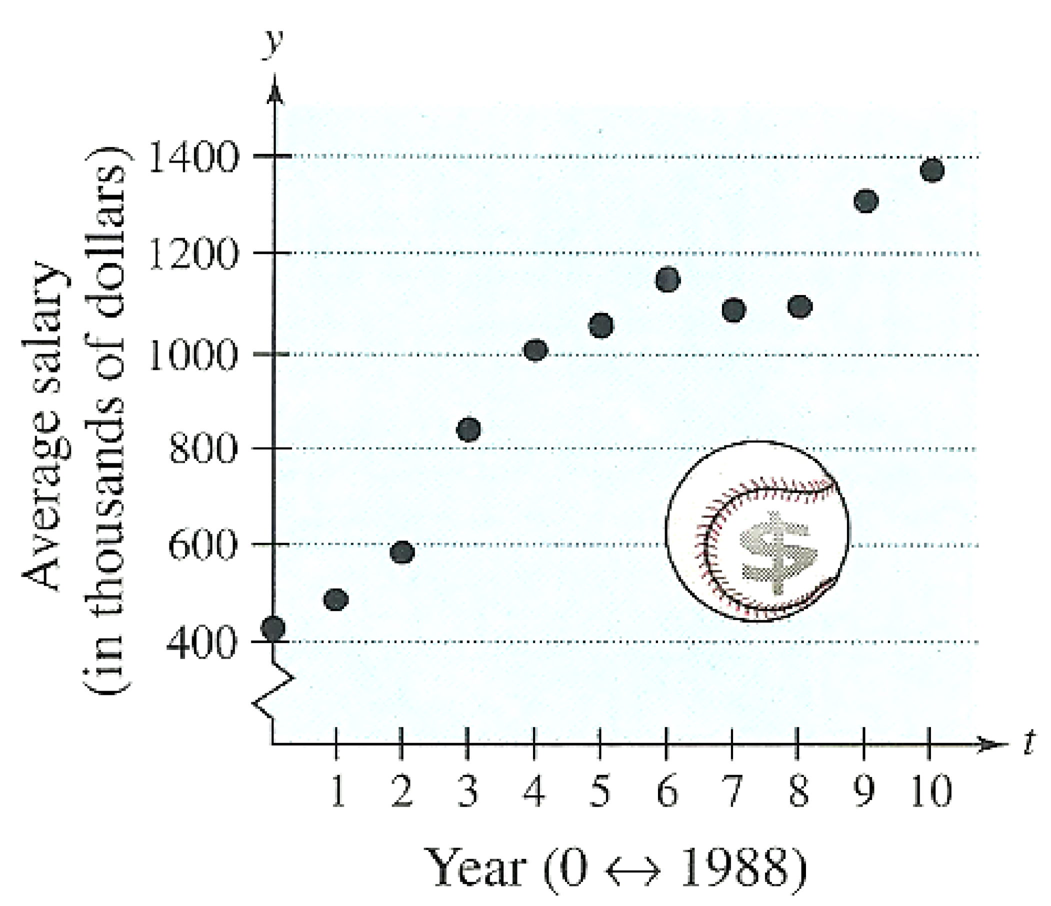
94. ***Real Estate Purchase*** A real estate office handles an apartment complex with 50 units. When the rent per unit is \$580 per month, all 50 units are occupied. However, when the rent is \$625 per month, the average number of occupied units drops to 47. Assume that the relationship between the monthly rent p and the demand x is linear.

(a) Write the equation of the line giving the demand x in terms of the rent p.

(b) Use a graphing utility to graph the demand equation and use the *trace* feature to estimate the number of units occupied if the rent is raised to \$655. Verify your answer algebraically.

(c) Use the demand equation to estimate the number of units occupied if the rent is lowered to \$595. Verify your answer graphically.

95. ***Sports*** The average annual salaries of Major League Baseball players (in thousands of dollars) from 1988 to 1998 are shown in the scatter plot. Let y represent the average salary and let t represent the year, with $t = 0$ corresponding to 1988. (Source: Major League Baseball Player Relations Committee)



(a) Use the regression capabilities of a graphing utility to find the line that best fits the data.

(b) Use the regression line to estimate the average salary in the year 2000.

(c) Interpret the meaning of the slope of the regression line.

Synthesis

True or False? **In Exercises 96 and 97, determine whether the statement is true or false. Justify your answer.**

96. The line through $(-8, 2)$ and $(-1, 4)$ and the line through $(0, -4)$ and $(-7, 7)$ are parallel.

97. If the points $(10, -3)$ and $(2, -9)$ lie on the same line, then the point $\left(-12, -\frac{37}{2}\right)$ also lies on that line.

98. ***Writing*** Explain how you could show that the points $A(2, 3)$, $B(2, 9)$, and $C(7, 3)$ are the vertices of a right triangle.

99. ***Think About It*** The slopes of two lines are -4 and $\frac{5}{2}$. Which is steeper?

100. ***Writing*** Write a brief paragraph explaining whether or not any pair of points on a line can be used to calculate the slope of the line.

101. ***Think About It*** Is it possible for two lines with positive slopes to be perpendicular? Explain.

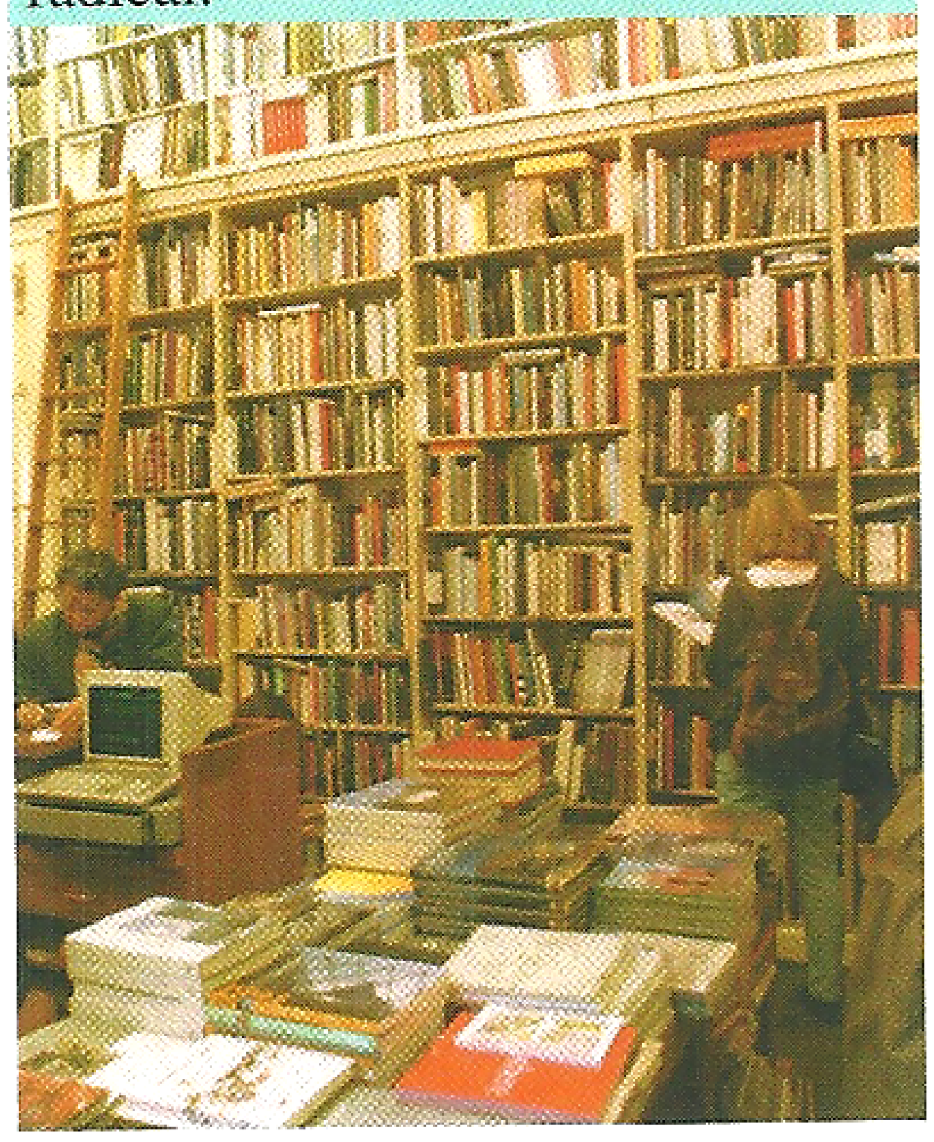
P.4 Solving Equations Algebraically and Graphically

What You Should Learn:

- How to solve linear equations
- How to find x- and y-intercepts of graphs of equations
- How to find solutions of equations graphically
- How to find the points of intersection of two graphs
- How to solve polynomial equations
- How to solve equations involving radicals, fractions, or absolute values

Why You Should Learn It:

Knowing how to solve equations graphically and algebraically can help you solve real-life problems. For instance, in Exercise 161 on page 53, you can find the number of copies of a book that will be sold at a certain price by solving a demand equation involving a radical.



Robert Holmes/CORBIS

Equations and Solutions of Equations

An **equation** is a statement that two algebraic expressions are equal. To **solve** an equation in x means to find all values of x for which the equation is true. Such values are **solutions.** For instance, $x = 4$ is a solution of the equation $3x - 5 = 7$, because $3(4) - 5 = 7$ is a true statement.

The solutions of an equation depend upon the kinds of numbers being considered. For instance, in the set of rational numbers, $x^2 = 10$ has no solution because there is no rational number whose square is 10. However, in the set of real numbers the equation has the two solutions $\sqrt{10}$ and $-\sqrt{10}$.

An equation that is true for every real number in the domain of the variable is called an **identity.** For example, $x^2 - 9 = (x + 3)(x - 3)$ is an identity because it is a true statement for any real value of x.

An equation that is true for just *some* (or even none) of the real numbers in the domain of the variable is called a **conditional equation.** For example, the equation $x^2 - 9 = 0$ is conditional because $x = 3$ and $x = -3$ are the only values in the domain that satisfy the equation. The equation $2x + 1 = 2x - 3$ is also conditional because it is not true for any values of x. A **linear equation in one variable x** is an equation that can be written in the standard form $ax + b = 0$, where a and b are real numbers with $a \neq 0$. For a review of solving one- and two-step equations, see Appendix C.

To solve an equation involving fractional expressions, find the least common denominator (LCD) of all terms in the equation and multiply every term by this LCD. This procedure clears the equation of fractions.

EXAMPLE 1 Solving an Equation Involving Fractions

$$\frac{x}{3} + \frac{3x}{4} = 2 \qquad \text{Original equation}$$

$$(12)\frac{x}{3} + (12)\frac{3x}{4} = (12)2 \qquad \text{Multiply by the LCD.}$$

$$4x + 9x = 24 \qquad \text{Simplify.}$$

$$13x = 24 \qquad \text{Combine like terms.}$$

$$x = \frac{24}{13} \qquad \text{Divide each side by 13.}$$

Check

After solving an equation, check the solution in the original equation.

$$\frac{\frac{24}{13}}{3} + \frac{3\left(\frac{24}{13}\right)}{4} \stackrel{?}{=} 2 \qquad \text{Substitute } \tfrac{24}{13} \text{ for } x.$$

$$2 = 2 \qquad \text{Solution checks. } \checkmark$$

When multiplying or dividing an equation by a *variable* expression, it is possible to introduce an **extraneous** solution—one that does not satisfy the original equation. The next example demonstrates the importance of checking your solution when you have multiplied or divided by a variable expression.

EXAMPLE 2 An Equation with an Extraneous Solution

Solve for x: $\dfrac{1}{x-2} = \dfrac{3}{x+2} - \dfrac{6x}{x^2-4}$

Algebraic Solution

In this case, the LCD is

$$x^2 - 4 = (x + 2)(x - 2).$$

Multiplying each term by the LCD and simplifying produces the following.

$$\frac{1}{x-2}(x+2)(x-2)$$

$$= \frac{3}{x+2}(x+2)(x-2) - \frac{6x}{x^2-4}(x+2)(x-2)$$

$$x + 2 = 3(x - 2) - 6x, \qquad x \neq \pm 2$$

$$x + 2 = 3x - 6 - 6x$$

$$4x = -8$$

$$x = -2$$

A check of $x = -2$ in the original equation shows that it yields a denominator of zero. So, $x = -2$ is extraneous, and the equation has *no solution.*

Graphical Solution

Use a graphing utility to graph the left and right sides of the equation

$$y_1 = \frac{1}{x-2} \quad \text{and} \quad y_2 = \frac{3}{x+2} - \frac{6x}{x^2-4}$$

in the same viewing window as shown in Figure P.39. The graphs of the equations do not appear to intersect. This means that there is no point for which the left side of the equation $1/(x - 2)$ is equal to the right side of the equation

$$\frac{3}{x+2} - \frac{6x}{x^2-4}.$$

So, the equation appears to have *no solution.*

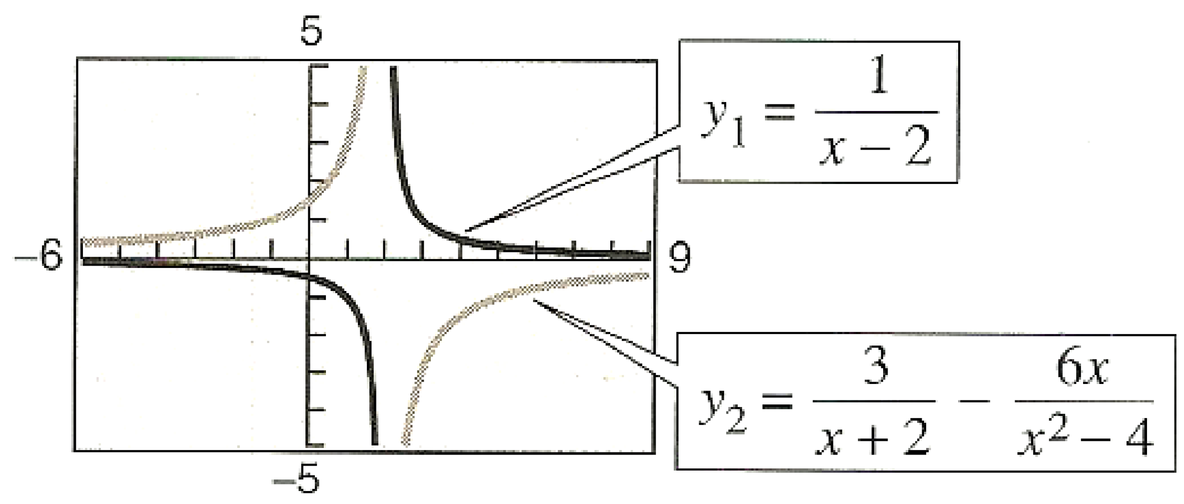


Figure P.39

Intercepts and Solutions

In Section P.2, you learned that the intercepts of a graph are the points at which the graph intersects the x- or y-axis.

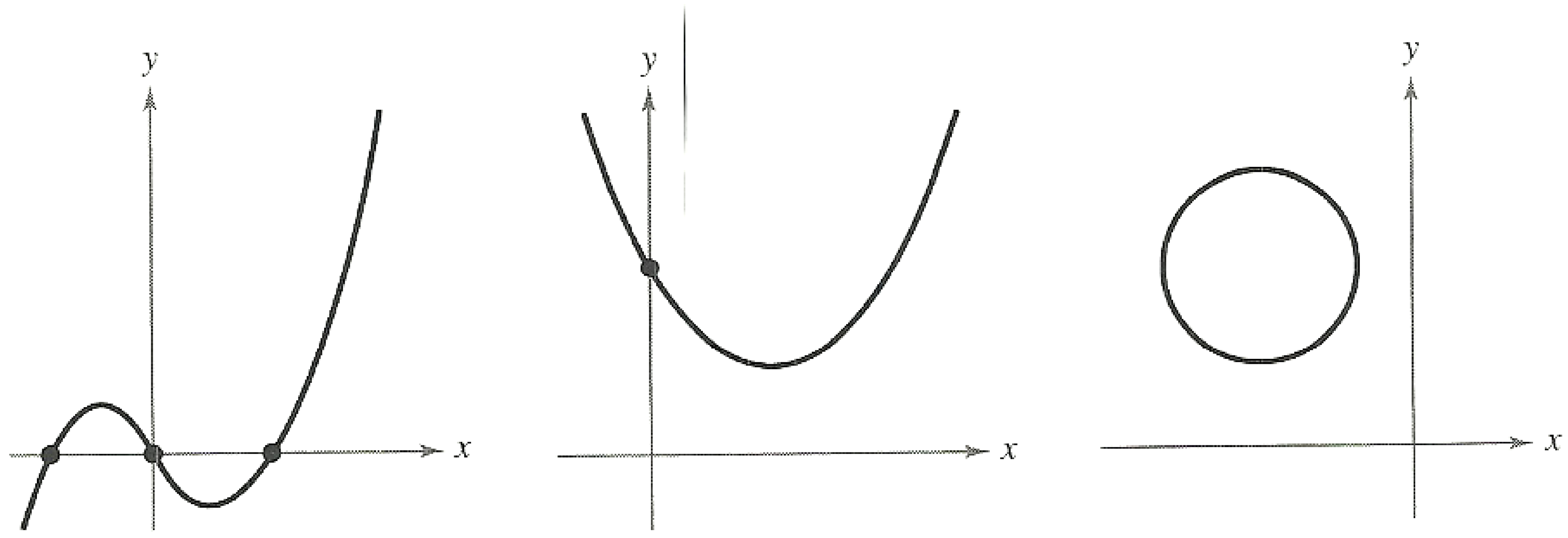
Definition of Intercepts

1. The point $(a, 0)$ is called an ***x*-intercept** of the graph of an equation if it is a solution point of the equation. To find the x-intercept(s), let $y = 0$ and solve the equation for x.
2. The point $(0, b)$ is called a ***y*-intercept** of the graph of an equation if it is a solution point of the equation. To find the y-intercept(s), let $x = 0$ and solve the equation for y.

An ancient Egyptian papyrus, discovered in 1858, contains one of the earliest examples of mathematical writing in existence. The papyrus itself dates back to around 1650 B.C., but it is actually a copy of writings from two centuries earlier. The algebraic equations on the papyrus were written in words. Diophantus, a Greek who lived around A.D. 250, is often called the Father of Algebra. He was the first to use abbreviated word forms in equations.

Sometimes it is convenient to denote the x-intercept as simply the x-coordinate of the point $(a, 0)$ rather than the point itself. Unless it is necessary to make a distinction, "intercept" will be used to mean either the point or the coordinate.

It is possible that a particular graph will have no intercepts or several intercepts. For instance, consider the three graphs in Figure P.40.



Three x-Intercepts
One y-Intercept

No x-Intercepts
One y-Intercept

No Intercepts

Figure P.40

Exploration

Use a graphing utility to graph each equation. How many times does each of the graphs intersect the x-axis and how many times does each intersect the y-axis?

$$y_1 = x^4 - 5x^2 + 4$$
$$y_2 = 3 + x^2$$
$$y_3 = x^2 - 3$$
$$y_4 = 2 \pm \sqrt{4x - x^2 - 3}$$

In general, how many times can an equation's graph intersect the x-axis? How many times can it intersect the y-axis?

EXAMPLE 3 Finding x- and y-Intercepts

Find the x- and y-intercepts of the graph of $2x + 3y = 5$.

Solution

To find the x-intercept, let $y = 0$. This produces

$$2x = 5 \quad \Rightarrow \quad x = \frac{5}{2}$$

which implies that the graph has one x-intercept: $\left(\frac{5}{2}, 0\right)$. To find the y-intercept, let $x = 0$. This produces

$$3y = 5 \quad \Rightarrow \quad y = \frac{5}{3}$$

which implies that the graph has one y-intercept: $\left(0, \frac{5}{3}\right)$. See Figure P.41.

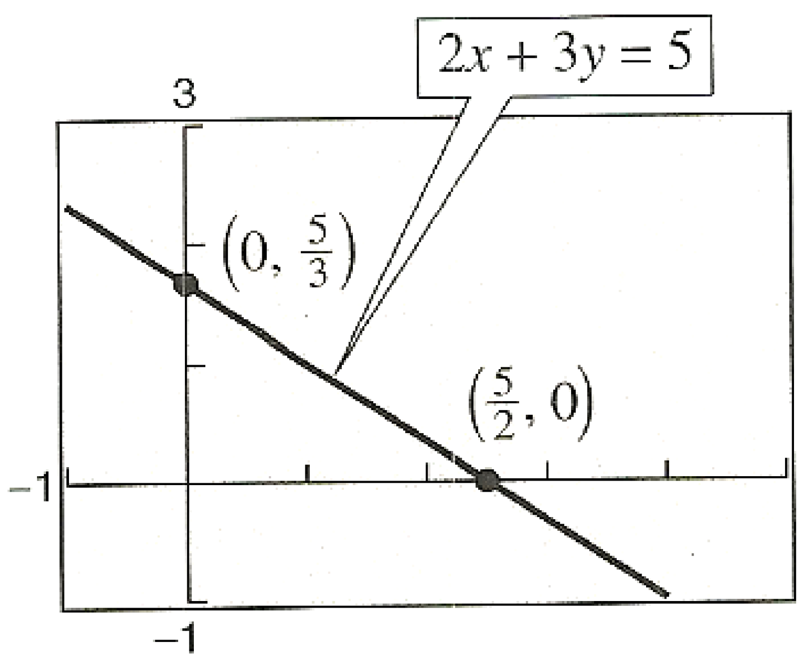


Figure P.41

The Interactive CD-ROM and Internet versions of this text offer a built-in graphing calculator, which can be used with the Examples, Explorations, and Exercises.

The concepts of x-intercepts and solutions of equations are closely related. In fact, the following statements are equivalent.

1. The point $(a, 0)$ is an *x-intercept* of the graph of an equation.
2. The number a is a *solution* of the equation $y = 0$.

This close connection among x-intercepts and solutions described on page 40 is crucial to our study of algebra, and you can take advantage of this connection in two basic ways. You can use your algebraic "equation-solving skills" to find the x-intercepts of a graph, and you can use your "graphing skills" to approximate the solutions of an equation.

Finding Solutions Graphically

Polynomial equations of degree 1 or 2 can be solved in relatively straightforward ways. Polynomial equations of higher degrees can, however, be quite difficult to solve, especially if you rely only on algebraic techniques. For such equations, a graphing utility can be very helpful.

Graphical Approximations of Solutions of an Equation

1. Write the equation in *general form,* $y = 0$, with the nonzero terms on one side of the equation and zero on the other side.
2. Use a graphing utility to graph the equation. Be sure the viewing window shows all the relevant features of the graph.
3. Use the *zero* or *root* feature or the *zoom* and *trace* features of the graphing utility to approximate each of the x-intercepts of the graph. Remember that a graph can have more than one x-intercept, so you may need to change the viewing window a few times.

In Chapter 2 you will learn techniques for determining the number of solutions of a polynomial equation. For now, you should know that a polynomial equation of degree n cannot have more than n different solutions.

Exploration

In Chapter 2 you will learn that a cubic equation such as

$$24x^3 - 36x + 17 = 0$$

can have up to 3 real solutions. Use a graphing utility to graph

$$y = 24x^3 - 36x + 17.$$

Describe a viewing window that allows you to determine the number of real solutions of the equation

$$24x^3 - 36x + 17 = 0.$$

Use the same technique to determine the number of real solutions of

$$97x^3 - 102x^2 - 200x - 63 = 0.$$

EXAMPLE 4 Finding Solutions of an Equation Graphically

Use a graphing utility to approximate the solutions of $2x^3 - 3x + 2 = 0$.

Solution

Begin by graphing the equation $y = 2x^3 - 3x + 2$, as shown in Figure P.42. You can see from the graph that there is only one x-intercept. It lies between -1 and -2 and is approximately -1.5. By using the *zero* or *root* feature of a graphing utility you can improve the approximation. To three-decimal-place accuracy, the solution is $x \approx -1.476$. Check this approximation on your calculator. You will find that the value of y is $y = 2(-1.476)^3 - 3(-1.476) + 2 \approx -0.003$.

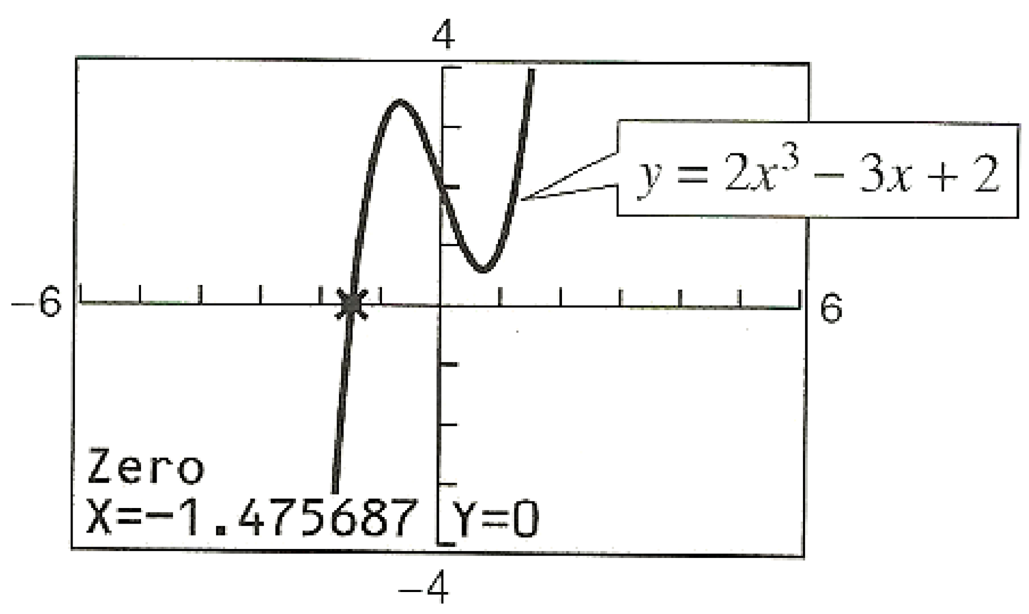


Figure P.42

You can also use a graphing calculator's *zoom* and *trace* features to approximate the solution of an equation. Here are some suggestions for using the *zoom-in* feature of a graphing utility.

1. With each successive zoom-in, adjust the x-scale (if necessary) so that the resulting viewing window shows at least the two scale marks between which the solution lies.
2. The accuracy of the approximation will always be such that the error is less than the distance between two scale marks.
3. If you have a *trace* feature on your graphing utility, you can generally add one more decimal place of accuracy without changing the viewing window.

Unless stated otherwise, this book will approximate all real solutions with an error of *at most* 0.01.

STUDY TIP

Remember that the more decimal places in the solution, the more accurate it is. You can reach the desired accuracy when zooming in as follows.

- To approximate the zero to the nearest hundredth, set the x-scale to 0.01.
- To approximate the zero to the nearest thousandth, set the x-scale to 0.001.

EXAMPLE 5 Approximating Solutions of an Equation Graphically

Use a graphing utility to approximate the solutions of $x^2 + 3 = 5x$.

Solution

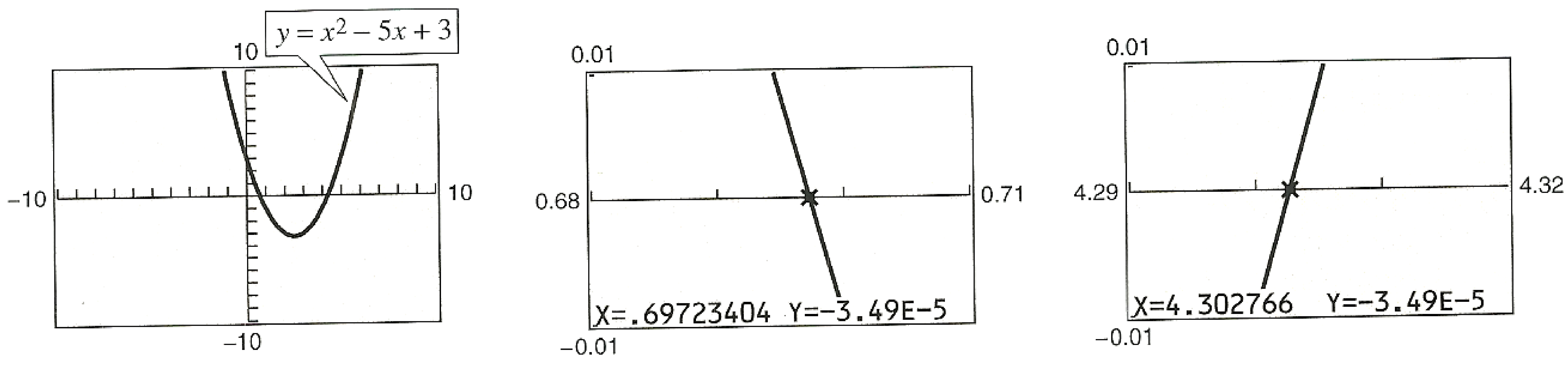
In general form, this equation is

$$x^2 - 5x + 3 = 0. \qquad \text{Equation in general form}$$

So, you can begin by graphing

$$y = x^2 - 5x + 3 \qquad \text{Equation to be graphed}$$

as shown in Figure P.43(a). This graph has two x-intercepts, and by using the *zoom* and *trace* features you can approximate the corresponding solutions to be $x \approx 0.70$ and $x \approx 4.30$, as shown in Figures P.43(b) and P.43(c).



(a) (b) (c)

Figure P.43

The built-in *zero* or *root* programs of a graphing utility will approximate solutions of equations or approximate x-intercepts of graphs. If your graphing utility has such features, try using them to approximate the solutions in Example 5.

Points of Intersection of Two Graphs

An ordered pair that is a solution of two different equations is called a **point of intersection** of the graphs of the two equations. For instance, in Figure P.44 you can see that the graphs of the following equations have two points of intersection.

$y = x + 2$ Equation 1

$y = x^2 - 2x - 2$ Equation 2

The point $(-1, 1)$ is a solution of both equations, and the point $(4, 6)$ is a solution of both equations. To check this algebraically, substitute -1 and 4 into each equation.

Check that $(-1, 1)$ *is a solution.*

Equation 1: $y = -1 + 2 = 1$ Solution checks. ✓

Equation 2: $y = (-1)^2 - 2(-1) - 2$

$= 1$ Solution checks. ✓

Check that $(4, 6)$ *is a solution.*

Equation 1: $y = 4 + 2$

$= 6$ Solution checks. ✓

Equation 2: $y = (4)^2 - 2(4) - 2$

$= 6$ Solution checks. ✓

To find the points of intersection of the graphs of two equations, solve each equation for y (or x) and set the two results equal to each other. The resulting equation will be an equation in one variable, which can be solved using standard procedures, as shown in Example 6.

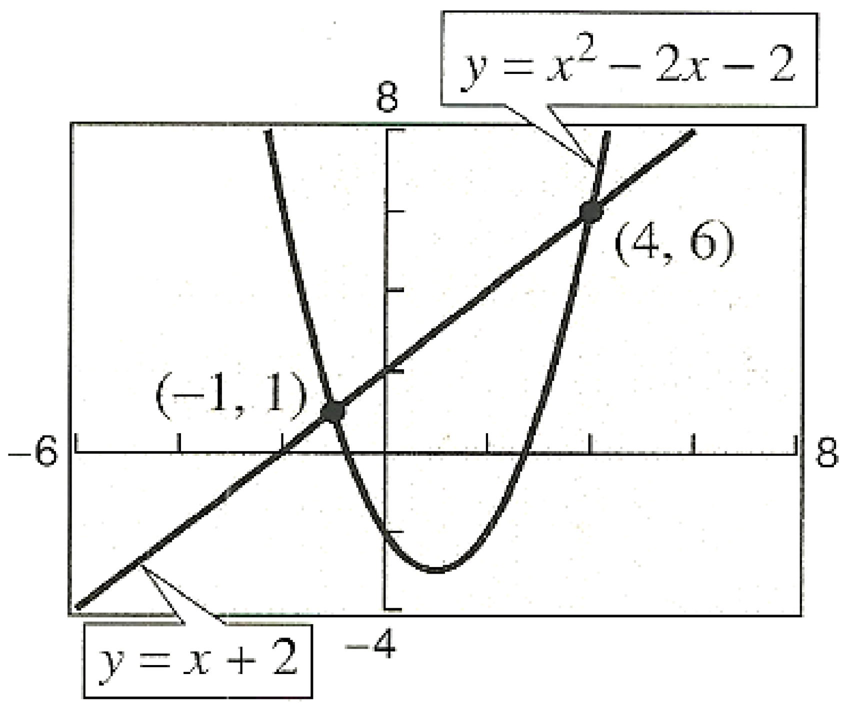


Figure P.44

STUDY TIP

The table shows some points of the graphs of the equations at the left. Find the points of intersection of the graphs by finding the values of x for which y_1 and y_2 are equal.

X	Y1	Y2
−2	0	6
−1	1	1
0	2	−2
1	3	−3
2	4	−2
3	5	1
4	6	6
X=−2		

EXAMPLE 6 Finding Points of Intersection

Find the points of intersection of the graphs of $2x - 3y = -2$ and $4x - y = 6$.

Algebraic Solution

To begin, solve each equation for y to obtain

$$y = \frac{2}{3}x + \frac{2}{3} \quad \text{and} \quad y = 4x - 6.$$

Next, set the two expressions for y equal to each other and solve the resulting equation for x, as follows.

$\frac{2}{3}x + \frac{2}{3} = 4x - 6$ Equate expressions for y.

$2x + 2 = 12x - 18$ Multiply each side by 3.

$-10x = -20$ Subtract $12x$ and 2 from each side.

$x = 2$ Divide each side by -10.

When $x = 2$, the y-value of each of the given equations is 2. So, the point of intersection is $(2, 2)$.

Graphical Solution

To begin, solve each equation for y to obtain $y_1 = \frac{2}{3}x + \frac{2}{3}$ and $y_2 = 4x - 6$. Then use a graphing utility to graph both equations in the same viewing window. In Figure P.45, the graphs appear to have one point of intersection. Use the *intersect* feature of the graphing utility to approximate the point of intersection to be $(2, 2)$.

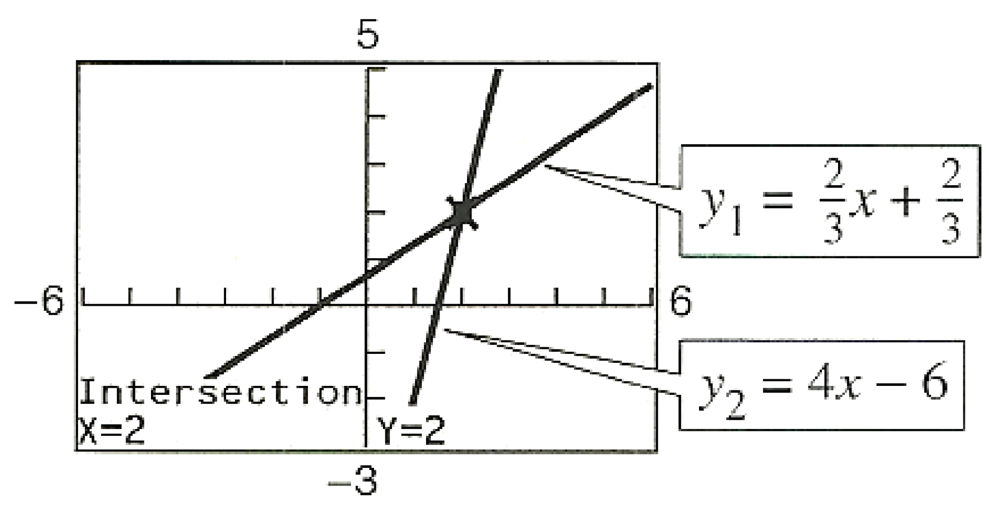


Figure P.45

Another way to approximate points of intersection of two graphs is to graph both equations and use the *zoom* and *trace* features to find the point or points at which the two graphs intersect.

EXAMPLE 7 Approximating Points of Intersection Graphically

Approximate the point(s) of intersection of the graphs of the following equations.

$y = x^2 - 3x - 4$ Equation 1 (quadratic equation)

$y = x^3 + 3x^2 - 2x - 1$ Equation 2 (cubic equation)

Solution

Begin by using a graphing utility to graph both equations, as shown in Figure P.46. From this display, you can see that the two graphs have only one point of intersection. Then, using the *zoom* and *trace* features, approximate the point of intersection to be $(-2.17, 7.25)$. To test the reasonableness of this approximation, you can evaluate both equations when $x = -2.17$.

Quadratic Equation:

$$y = (-2.17)^2 - 3(-2.17) - 4$$
$$\approx 7.22$$

Cubic Equation:

$$y = (-2.17)^3 + 3(-2.17)^2 - 2(-2.17) - 1$$
$$\approx 7.25$$

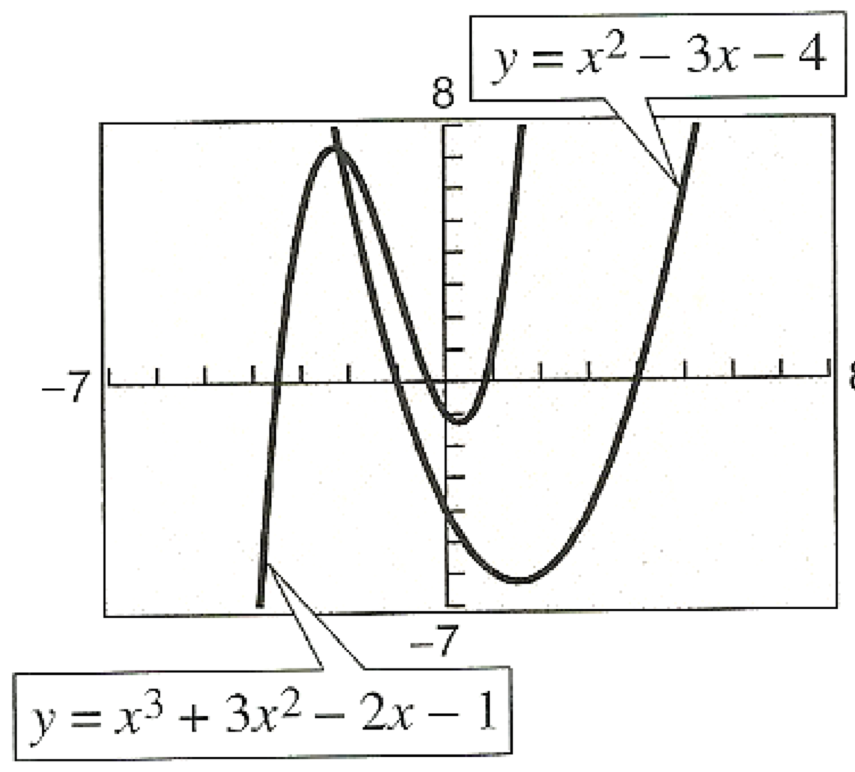


Figure P.46

Because both equations yield approximately the same y-value, you can conclude that the approximate coordinates of the point of intersection are $x \approx -2.17$ and $y \approx 7.25$.

The method shown in Example 7 gives a nice graphical picture of the points of intersection of two graphs. However, for actual approximation purposes, it is better to use the algebraic procedure described in Example 6. That is, the point of intersection of $y = x^2 - 3x - 4$ and $y = x^3 + 3x^2 - 2x - 1$ coincides with the solution of the equation

$x^3 + 3x^2 - 2x - 1 = x^2 - 3x - 4$ Equate y-values.

$x^3 + 2x^2 + x + 3 = 0.$ Write in general form.

By graphing $y = x^3 + 2x^2 + x + 3$ on a graphing utility and using the *zoom* and *trace* features (or the *zero* or *root* feature), you can approximate the solution of this equation to be $x \approx -2.17$. The corresponding y-value for *both* of the equations given in Example 7 is $y \approx 7.25$.

Solving Polynomial Equations Algebraically

Polynomial equations can be classified by their degree.

Degree	*Name*	*Example*
First degree	Linear equation	$6x + 2 = 4$
Second degree	Quadratic equation	$2x^2 - 5x + 3 = 0$
Third degree	Cubic equation	$x^3 - x = 0$
Fourth degree	Quartic equation	$x^4 - 3x^2 + 2 = 0$
Fifth degree	Quintic equation	$x^5 - 12x^2 + 7x + 4 = 0$

In general, the higher the degree, the more difficult it is to solve the equation—either algebraically or graphically.

You should be familiar with the following four methods for solving quadratic equations *algebraically*.

Solving a Quadratic Equation

Method

Factoring: If $ab = 0$, then $a = 0$ or $b = 0$.

Example

$$x^2 - x - 6 = 0$$
$$(x - 3)(x + 2) = 0$$
$$x - 3 = 0 \Rightarrow x = 3$$
$$x + 2 = 0 \Rightarrow x = -2$$

Square Root Principle: If $u^2 = c$, where $c > 0$, then $u = \pm\sqrt{c}$.

$$(x + 3)^2 = 16$$
$$x + 3 = \pm 4$$
$$x = -3 \pm 4$$
$$x = 1 \quad \text{or} \quad x = -7$$

Completing the Square: If $x^2 + bx = c$, then

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2$$
$$\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}.$$

$$x^2 + 6x = 5$$
$$x^2 + 6x + 3^2 = 5 + 3^2$$
$$(x + 3)^2 = 14$$
$$x + 3 = \pm\sqrt{14}$$
$$x = -3 \pm \sqrt{14}$$

Quadratic Formula: If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

$$2x^2 + 3x - 1 = 0$$
$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$
$$= \frac{-3 \pm \sqrt{17}}{4}$$

The methods used to solve quadratic equations can sometimes be extended to polynomial equations of higher degree, as shown in the next two examples.

EXAMPLE 8 Solving an Equation of Quadratic Type

Solve the equation $x^4 - 3x^2 + 2 = 0$.

Solution

The expression $x^4 - 3x^2 + 2$ is said to be in *quadratic form* because it is written in the form $au^2 + bu + c$, where u is any expression in x, namely x^2. You can use factoring to solve the equation as follows.

$$x^4 - 3x^2 + 2 = 0 \qquad \text{Write original equation.}$$

$$(x^2)^2 - 3(x^2) + 2 = 0 \qquad \text{Write in quadratic form in } x^2.$$

$$(x^2 - 1)(x^2 - 2) = 0 \qquad \text{Partially factor.}$$

$$(x + 1)(x - 1)(x^2 - 2) = 0 \qquad \text{Factor.}$$

$$x + 1 = 0 \quad \Rightarrow \quad x = -1$$

$$x - 1 = 0 \quad \Rightarrow \quad x = 1$$

$$x^2 - 2 = 0 \quad \Rightarrow \quad x = \pm\sqrt{2}$$

The equation has four solutions: -1, 1, $\sqrt{2}$, and $-\sqrt{2}$. Check these solutions in the original equation. The graph of $y = x^4 - 3x^2 + 2$, as shown in Figure P.47, verifies the solutions graphically.

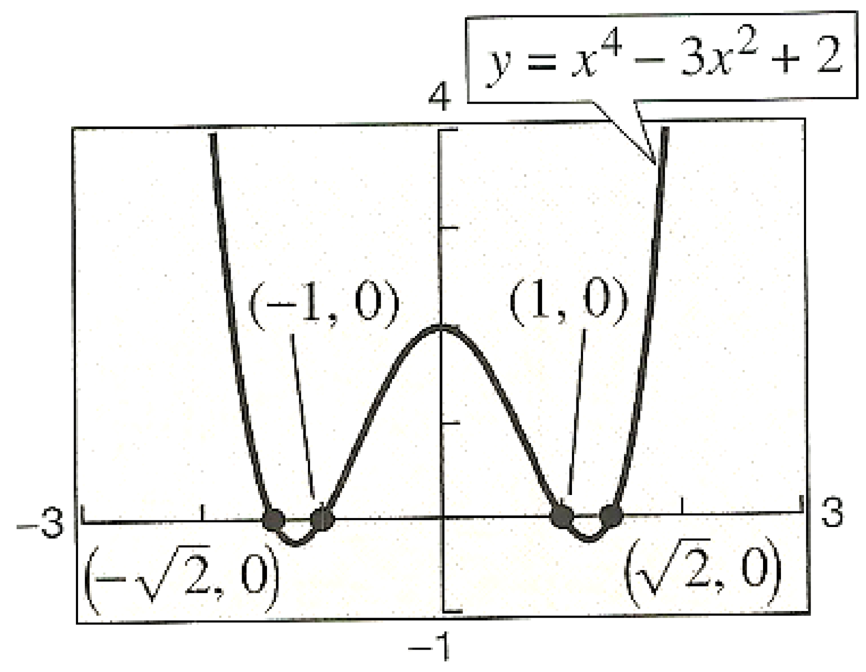


Figure P.47

EXAMPLE 9 Solving a Polynomial Equation by Factoring

Solve the equation $2x^3 - 6x^2 - 6x + 18 = 0$.

Solution

This equation has a common factor of 2. To simplify things, first divide each side of the equation by 2.

$$2x^3 - 6x^2 - 6x + 18 = 0 \qquad \text{Write original equation.}$$

$$x^3 - 3x^2 - 3x + 9 = 0 \qquad \text{Divide each side by 2.}$$

$$x^2(x - 3) - 3(x - 3) = 0 \qquad \text{Factor out common monomial factors.}$$

$$(x - 3)(x^2 - 3) = 0 \qquad \text{Factor by grouping.}$$

$$x - 3 = 0 \quad \Rightarrow \quad x = 3$$

$$x^2 - 3 = 0 \quad \Rightarrow \quad x = \pm\sqrt{3}$$

The equation has three solutions: 3, $\sqrt{3}$, and $-\sqrt{3}$. Check these solutions in the original equation. The graph of $y = 2x^3 - 6x^2 - 6x + 18$, as shown in Figure P.48, verifies the solutions graphically.

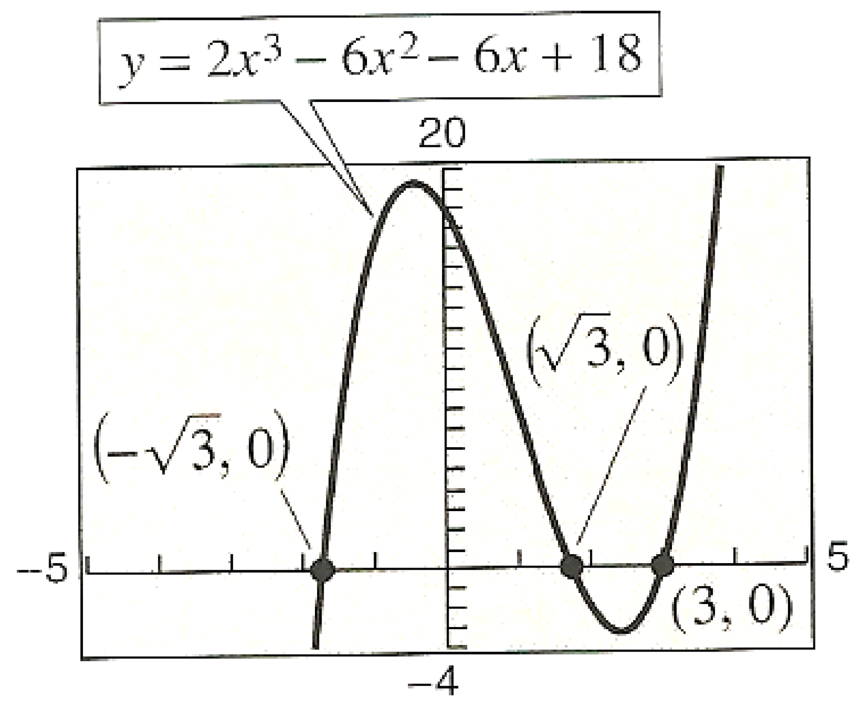


Figure P.48

Other Types of Equations

An equation involving a radical expression can often be cleared of radicals by raising both sides of the equation to an appropriate power. When using this procedure, remember that it can introduce extraneous solutions—so be sure to check each solution in the original equation.

EXAMPLE 10 Solving an Equation Involving a Radical

Solve $\sqrt{2x+7} - x = 2$.

Algebraic Solution

$\sqrt{2x+7} - x = 2$ Write original equation.

$\sqrt{2x+7} = x + 2$ Isolate radical.

$2x + 7 = x^2 + 4x + 4$ Square each side.

$x^2 + 2x - 3 = 0$ Write in general form.

$(x+3)(x-1) = 0$ Factor.

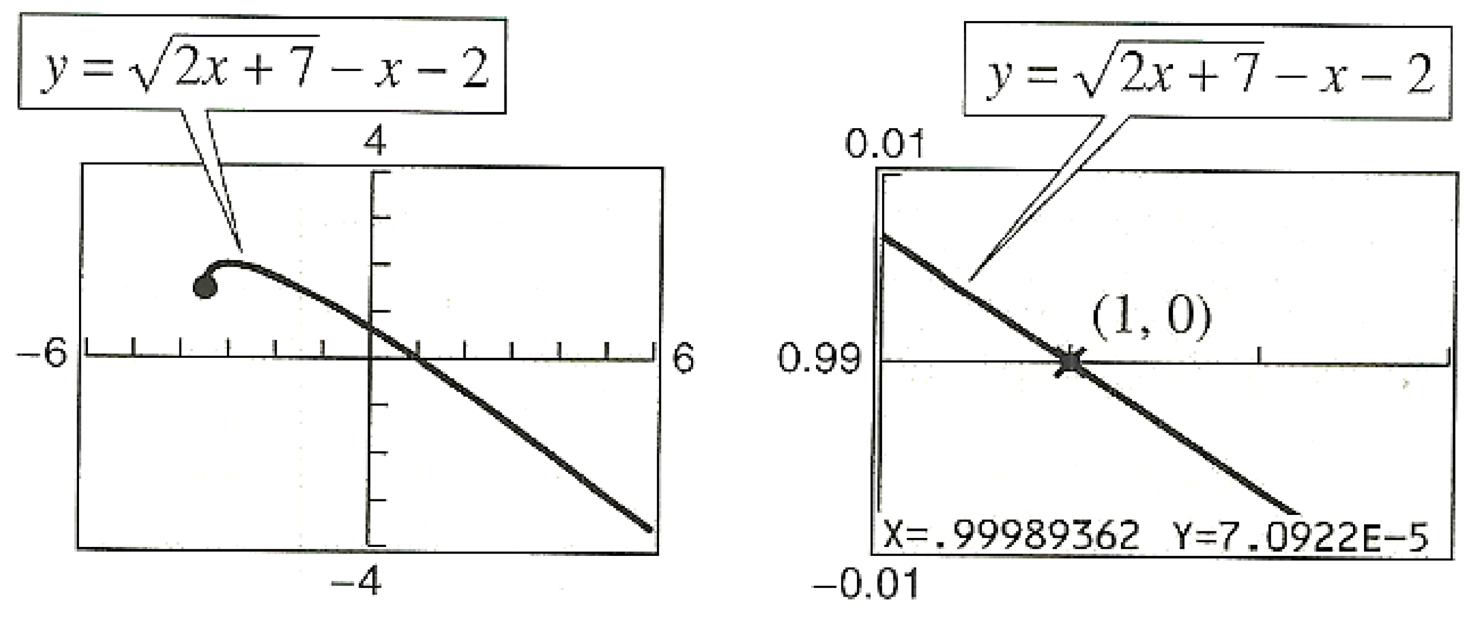
$x + 3 = 0 \Rightarrow x = -3$ Set 1st factor equal to 0.

$x - 1 = 0 \Rightarrow x = 1$ Set 2nd factor equal to 0.

By substituting into the original equation, you can determine that -3 is extraneous, whereas 1 is valid. So, the equation has only one real solution: $x = 1$.

Graphical Solution

First rewrite the equation as $\sqrt{2x+7} - x - 2 = 0$. Then use a graphing utility to graph $y = \sqrt{2x+7} - x - 2$, as shown in Figure P.49(a). Notice that the domain is $x \geq -\frac{7}{2}$ because the expression under the radical cannot be negative. There appears to be one solution near $x = 1$. Use the *zoom* and *trace* features, as shown in Figure P.49(b), to approximate the only solution to be $x = 1$.



(a) (b)

Figure P.49

EXAMPLE 11 Solving an Equation Involving Two Radicals

Solve the equation $\sqrt{2x+6} - \sqrt{x+4} = 1$.

Solution

$\sqrt{2x+6} - \sqrt{x+4} = 1$ Write original equation.

$2x + 6 = 1 + 2\sqrt{x+4} + (x+4)$ Isolate radical and square each side.

$x + 1 = 2\sqrt{x+4}$ Isolate radical.

$x^2 + 2x + 1 = 4(x+4)$ Square each side.

$x^2 - 2x - 15 = 0$ Write in general form.

$(x-5)(x+3) = 0$ Factor.

$x - 5 = 0 \Rightarrow x = 5$ Set 1st factor equal to 0.

$x + 3 = 0 \Rightarrow x = -3$ Set 2nd factor equal to 0.

By substituting into the original equation, you can determine that -3 is extraneous, whereas 5 is valid. Figure P.50 verifies that $x = 5$ is the only solution.

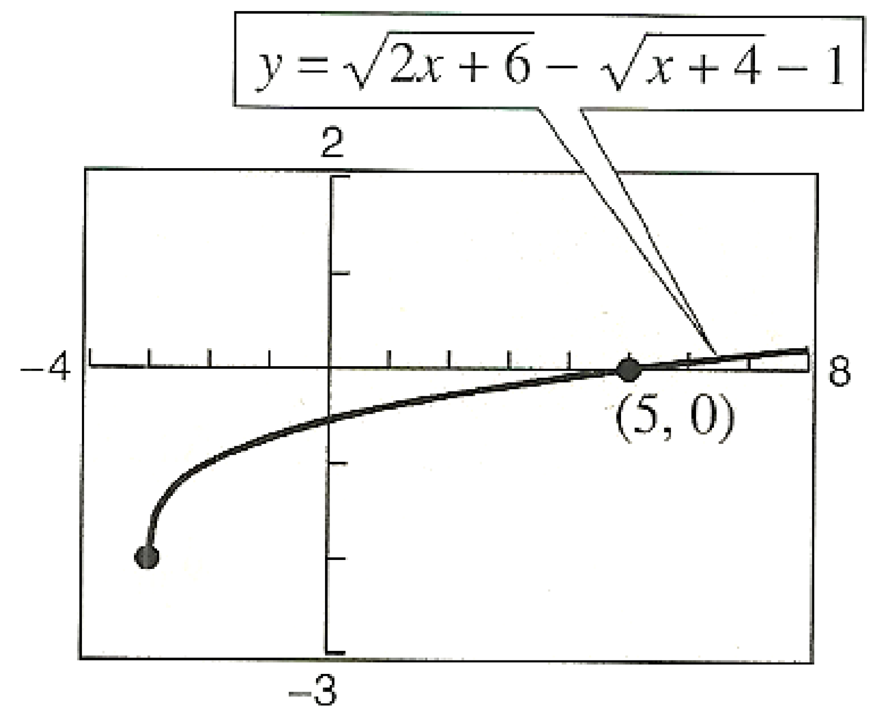


Figure P.50

Sometimes radicals in equations are represented by fractional exponents.

EXAMPLE 12 Solving an Equation with Rational Exponents

Solve $(x + 1)^{2/3} = 4$.

Algebraic Solution

$$(x + 1)^{2/3} = 4 \qquad \text{Write original equation.}$$

$$\sqrt[3]{(x + 1)^2} = 4 \qquad \text{Rewrite with radical sign.}$$

$$(x + 1)^2 = 64 \qquad \text{Cube each side.}$$

$$x + 1 = \pm 8 \qquad \text{Take square root of each side.}$$

$$x = -9, x = 7 \qquad \text{Subtract 1 from each side.}$$

Substitute $x = -9$ and $x = 7$ into the original equation to determine that both are valid solutions.

Graphical Solution

Use a graphing utility to graph $y_1 = \sqrt[3]{(x + 1)^2}$ and $y_2 = 4$ in the same viewing window, as shown in Figure P.51. Use the *intersect* feature of the graphing utility to approximate the solutions to be $x = -9$ and $x = 7$.

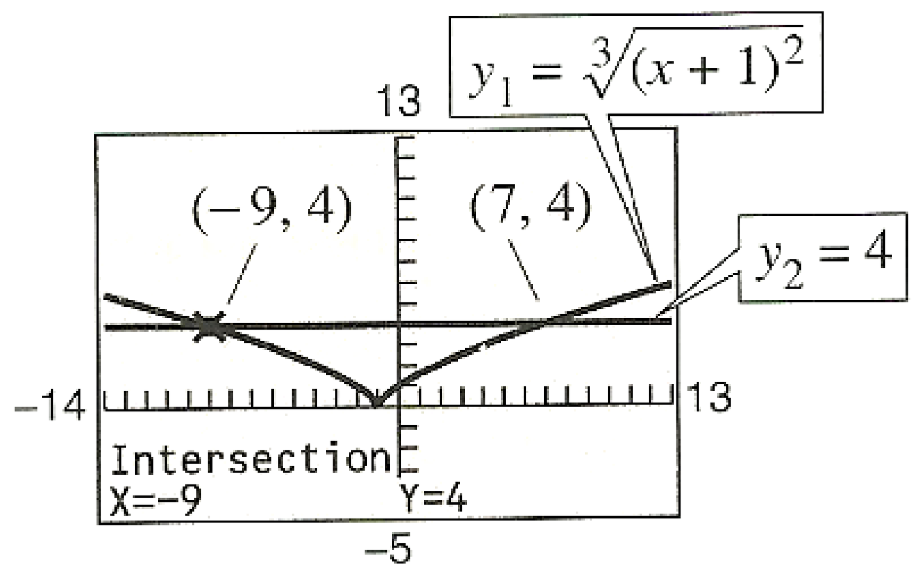


Figure P.51

As demonstrated in Example 1, you can algebraically solve an equation involving fractions by multiplying both sides of the equation by the least common denominator of each term in the equation to "clear an equation of fractions."

EXAMPLE 13 Solving an Equation Involving Fractions

Solve $\dfrac{2}{x} = \dfrac{3}{x - 2} - 1$.

Solution

For this equation, the least common denominator of the three terms is $x(x - 2)$, so you can begin by multiplying each term in the equation by this expression.

$$\frac{2}{x} = \frac{3}{x - 2} - 1$$

$$x(x - 2)\frac{2}{x} = x(x - 2)\frac{3}{x - 2} - x(x - 2)(1)$$

$$2(x - 2) = 3x - x(x - 2), \qquad x \neq 0, 2$$

$$2x - 4 = -x^2 + 5x$$

$$x^2 - 3x - 4 = 0$$

$$(x - 4)(x + 1) = 0$$

$$x - 4 = 0 \quad \Rightarrow \quad x = 4$$

$$x + 1 = 0 \quad \Rightarrow \quad x = -1$$

The equation has two solutions: 4 and -1. Check these solutions in the original equation. Use a graphing utility to verify these solutions graphically.

Exploration

Using *dot* mode, graph the two equations

$$y_1 = \frac{2}{x}$$

$$y_2 = \frac{3}{x - 2} - 1$$

in the same viewing window. How many times do the graphs of the equations intersect? What does this tell you about the solution to Example 13?

STUDY TIP

Graphs of equations involving variable denominators can be tricky because of the way graphing utilities skip over points where the denominator is zero. You will study graphs of such equations in Sections 2.6 and 2.7.

EXAMPLE 14 Solving an Equation Involving Absolute Value

Solve $|x^2 - 3x| = -4x + 6$.

Solution

Begin by writing the equation as $|x^2 - 3x| + 4x - 6 = 0$. From the graph of $y = |x^2 - 3x| + 4x - 6$ in Figure P.52, you can estimate the solutions to be -3 and 1. These can be verified by substitution into the equation. To solve an equation involving an absolute value *algebraically*, you must consider the fact that the expression inside the absolute value symbols can be positive or negative. This consideration results in *two* separate equations, each of which must be solved.

First Equation:

$x^2 - 3x = -4x + 6$ Use positive expression.

$x^2 + x - 6 = 0$ Write in general form.

$(x + 3)(x - 2) = 0$ Factor.

$x + 3 = 0 \Rightarrow x = -3$ Set 1st factor equal to 0.

$x - 2 = 0 \Rightarrow x = 2$ Set 2nd factor equal to 0.

Second Equation:

$-(x^2 - 3x) = -4x + 6$ Use negative expression.

$x^2 - 7x + 6 = 0$ Write in general form.

$(x - 1)(x - 6) = 0$ Factor.

$x - 1 = 0 \Rightarrow x = 1$ Set 1st factor equal to 0.

$x - 6 = 0 \Rightarrow x = 6$ Set 2nd factor equal to 0.

Check

$|(-3)^2 - 3(-3)| \stackrel{?}{=} -4(-3) + 6$ Substitute -3 for x.

$18 = 18$ -3 checks. ✓

$|2^2 - 3(2)| \stackrel{?}{=} -4(2) + 6$ Substitute 2 for x.

$2 \neq -2$ 2 does not check.

$|1^2 - 3(1)| \stackrel{?}{=} -4(1) + 6$ Substitute 1 for x.

$2 = 2$ 1 checks. ✓

$|6^2 - 3(6)| \stackrel{?}{=} -4(6) + 6$ Substitute 6 for x.

$18 \neq -18$ 6 does not check.

The equation has only two solutions: -3 and 1, just as you obtained by graphing.

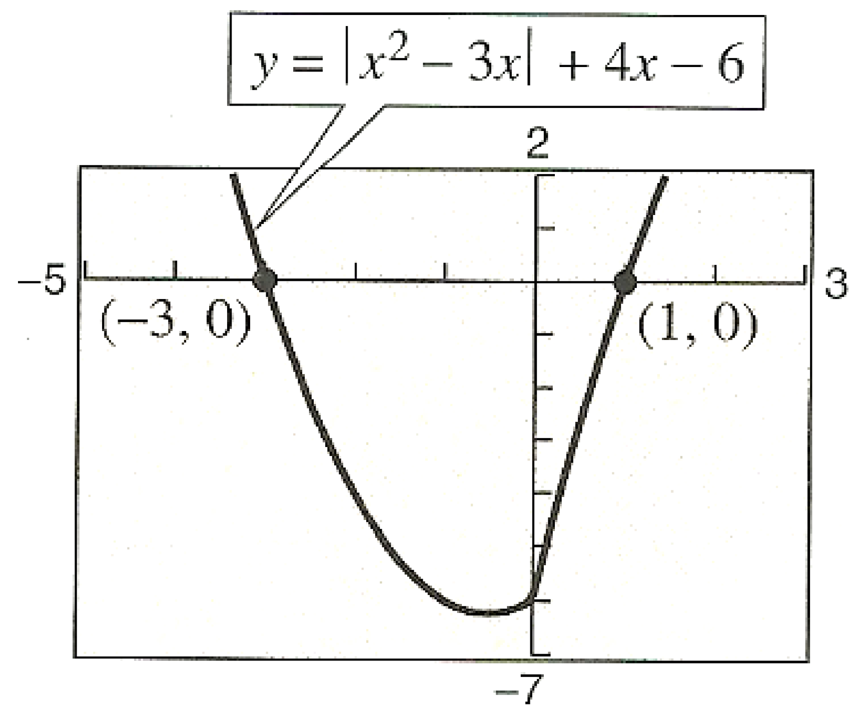


Figure P.52

In Figure P.52, the graph of $y = |x^2 - 3x| + 4x - 6$ appears to be a straight line to the right of the y-axis. Is it? Explain how you decided.

P.4 Exercises

In Exercises 1–6, determine whether the given values of x are solutions of the equation.

Equation	Values	
1. $\frac{5}{2x} - \frac{4}{x} = 3$	(a) $x = -\frac{1}{2}$	(b) $x = 4$
	(c) $x = 0$	(d) $x = \frac{1}{4}$
2. $\frac{x}{2} + \frac{6x}{7} = \frac{19}{14}$	(a) $x = -2$	(b) $x = 1$
	(c) $x = \frac{1}{2}$	(d) $x = 7$
3. $3 + \frac{1}{x+2} = 4$	(a) $x = -1$	(b) $x = -2$
	(c) $x = 0$	(d) $x = 5$
4. $\frac{(x+5)(x-3)}{2} = 24$	(a) $x = -3$	(b) $x = -2$
	(c) $x = 7$	(d) $x = 9$
5. $\frac{\sqrt{x+4}}{6} + 3 = 4$	(a) $x = -3$	(b) $x = 0$
	(c) $x = 21$	(d) $x = 32$
6. $\frac{\sqrt[3]{x-8}}{3} = -\frac{2}{3}$	(a) $x = -16$	(b) $x = 0$
	(c) $x = 9$	(d) $x = 16$

In Exercises 7–12, determine whether the equation is an identity or a conditional equation.

7. $2(x-1) = 2x - 2$
8. $-7(x-3) + 4x = 3(7-x)$
9. $x^2 - 8x + 5 = (x-4)^2 - 11$
10. $x^2 + 2(3x-2) = x^2 + 6x - 4$
11. $3 + \frac{1}{x+1} = \frac{4x}{x+1}$
12. $\frac{5}{x} + \frac{3}{x} = 24$

In Exercises 13 and 14, solve the equation in two ways. Then explain which way was easier for you.

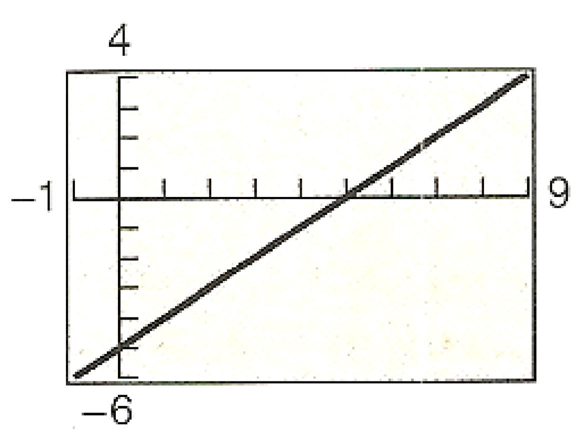
13. $\frac{3x}{8} - \frac{4x}{3} = 4$
14. $\frac{3z}{8} - \frac{z}{10} = 6$

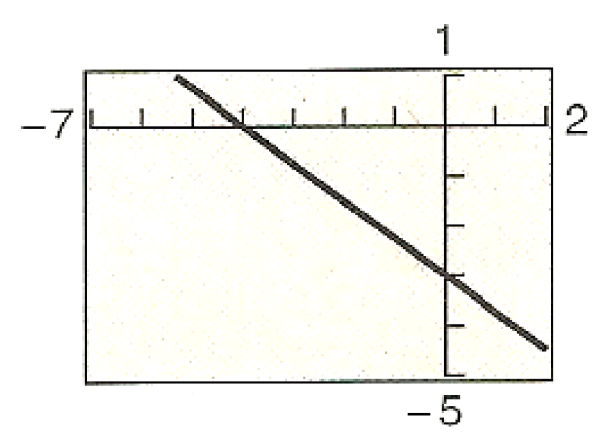
In Exercises 15–30, solve the equation (if possible). Then use a graphing utility to verify your solution.

15. $\frac{5x}{4} + \frac{1}{2} = x - \frac{1}{2}$
16. $\frac{x}{5} - \frac{x}{2} = 3$
17. $\frac{3}{2}(z+5) - \frac{1}{4}(z+24) = 0$
18. $\frac{3x}{2} + \frac{1}{4}(x-2) = 10$
19. $\frac{100-4u}{3} = \frac{5u+6}{4} + 6$
20. $\frac{17+y}{y} + \frac{32+y}{y} = 100$
21. $\frac{5x-4}{5x+4} = \frac{2}{3}$
22. $\frac{15}{x} - 4 = \frac{6}{x} + 3$
23. $\frac{1}{x-3} + \frac{1}{x+3} = \frac{10}{x^2-9}$
24. $\frac{1}{x-2} + \frac{3}{x+3} = \frac{4}{x^2+x-6}$
25. $\frac{7}{2x+1} - \frac{8x}{2x-1} = -4$
26. $\frac{4}{u-1} + \frac{6}{3u+1} = \frac{15}{3u+1}$
27. $\frac{1}{x} + \frac{2}{x-5} = 0$
28. $\frac{6}{x} - \frac{2}{x+3} = \frac{3(x+5)}{x(x+3)}$
29. $\frac{3}{x(x-3)} + \frac{4}{x} = \frac{1}{x-3}$
30. $3 = 2 + \frac{2}{z+2}$

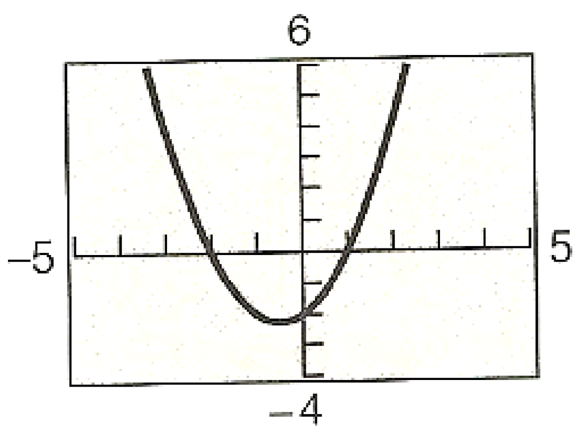
In Exercises 31–40, find the x- and y-intercepts of the graph of the equation.

31. $y = x - 5$
32. $y = -\frac{3}{4}x - 3$

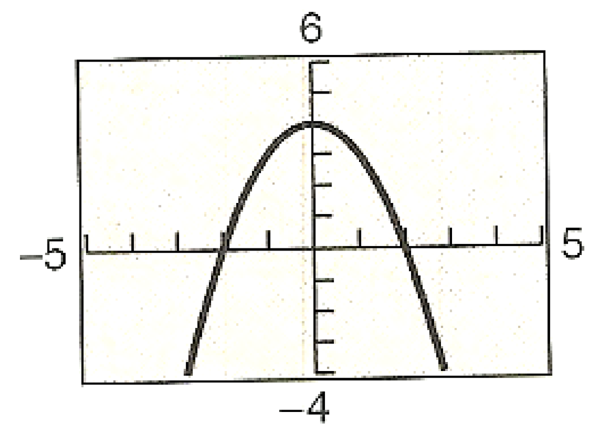




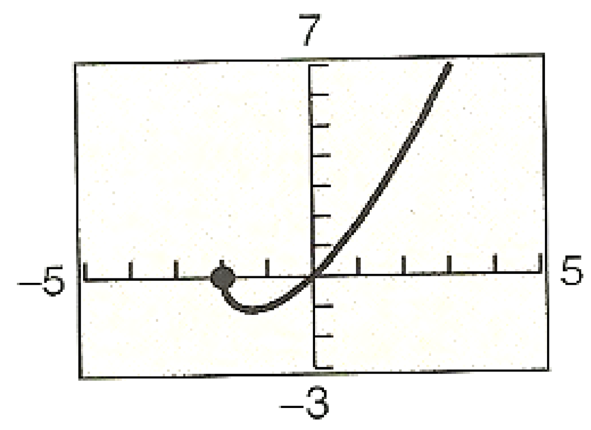
33. $y = x^2 + x - 2$



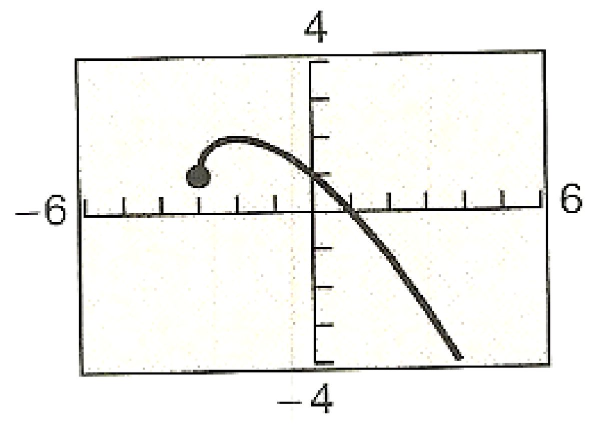
34. $y = 4 - x^2$



35. $y = x\sqrt{x+2}$



36. $y = -\frac{1}{2}x\sqrt{x+3} + 1$



37. $y = |x - 2| - 4$ **38.** $y = 3 - \frac{1}{2}|x + 1|$

39. $xy - 2y - x + 1 = 0$

40. $x^2y - x^2 + 4y = 0$

In Exercises 41–46, the solution(s) of the equation are given. Verify the solution(s) both algebraically and graphically.

Equation	*Solution(s)*
41. $y = 12 - 4x$	$x = 3$
42. $y = 3(x - 5) + 9$	$x = 2$
43. $y = x^2 - 2.5x - 6$	$x = -1.5, 4$
44. $y = x^3 - 9x^2 + 18x$	$x = 0, 3, 6$
45. $y = \dfrac{x+2}{3} - \dfrac{x-1}{5} - 1$	$x = 1$
46. $y = x - 3 - \dfrac{10}{x}$	$x = -2, 5$

Graphical Analysis **In Exercises 47–50, use a graphing utility to graph the equation and approximate any x-intercepts. Set $y = 0$ and solve the resulting equation. Compare the results with the x-intercepts of the graph.**

47. $y = 2(x - 1) - 4$ **48.** $y = \frac{4}{3}x + 2$

49. $y = 20 - (3x - 10)$ **50.** $y = 10 + 2(x - 2)$

In Exercises 51–70, solve the equation algebraically. Then write the equation in the form $y = 0$ and use a graphing utility to verify the algebraic solution.

51. $27 - 4x = 12$ **52.** $3.5x - 8 = 0.5x$

53. $25(x - 3) = 12(x + 2) - 10$

54. $1200 = 300 + 2(x - 500)$

55. $\dfrac{3x}{2} + \dfrac{1}{4}(x - 2) = 10$

56. $0.60x + 0.40(100 - x) = 50$

57. $\dfrac{2x}{3} = 10 - \dfrac{24}{x}$ **58.** $\dfrac{x-3}{25} = \dfrac{x-5}{12}$

59. $\dfrac{3}{x+2} - \dfrac{4}{x-2} = 5$ **60.** $\dfrac{6}{x} + \dfrac{8}{x+5} = 3$

61. $3(x + 3) = 5(1 - x) - 1$

62. $(x + 1)^2 + 2(x - 2) = (x + 1)(x - 2)$

63. $2x^3 - x^2 - 18x + 9 = 0$

64. $4x^3 + 12x^2 - 26x - 24 = 0$

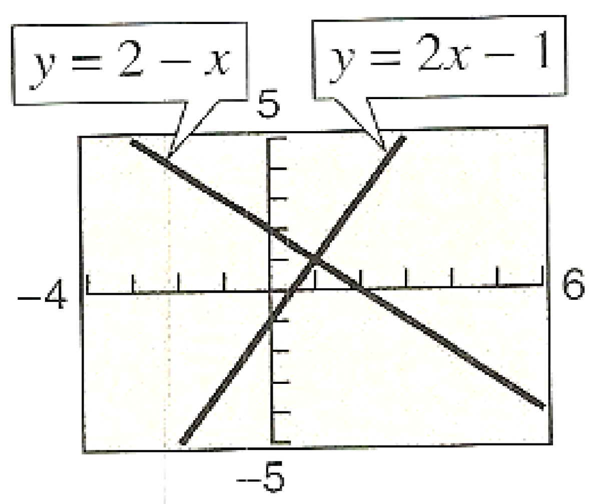
65. $x^4 = 2x^3 + 1$ **66.** $x^5 = 3 + 2x^3$

67. $\dfrac{2}{x+2} = 3$ **68.** $\dfrac{5}{x} = 1 + \dfrac{3}{x+2}$

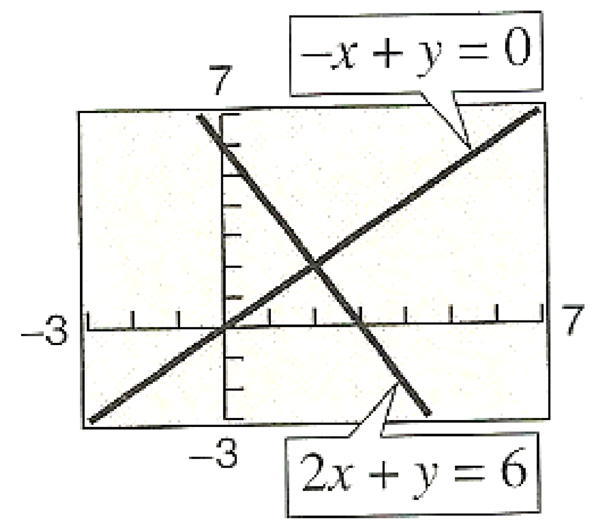
69. $|x - 3| = 4$ **70.** $\sqrt{x-2} = 3$

In Exercises 71–74, determine any point(s) of intersection algebraically. Then verify your result numerically by creating a table of values for each equation.

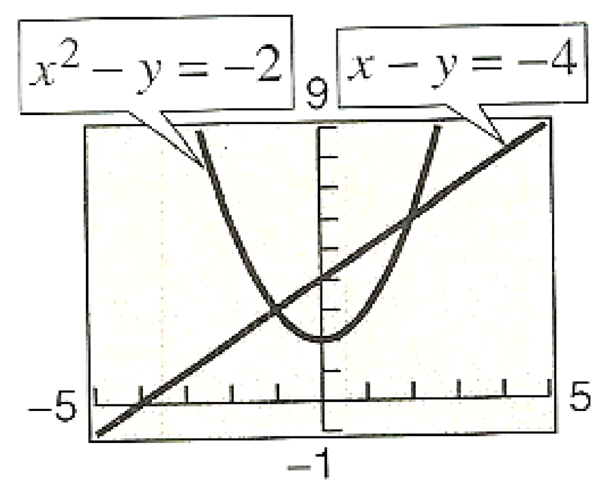
71. $y = 2 - x$
$y = 2x - 1$



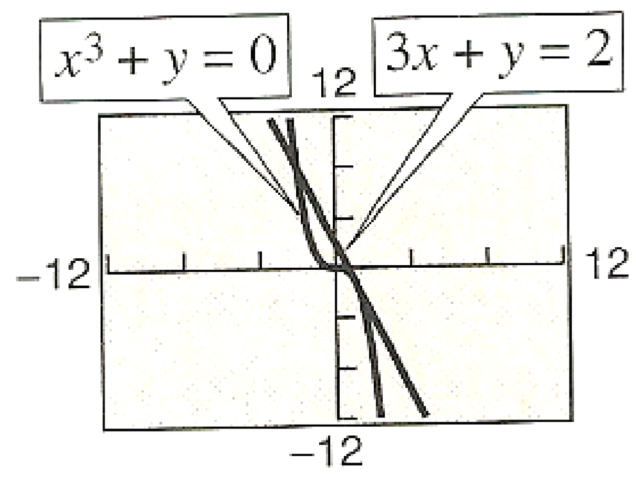
72. $2x + y = 6$
$-x + y = 0$



73. $x - y = -4$
$x^2 - y = -2$



74. $3x + y = 2$
$x^3 + y = 0$



In Exercises 75–80, use a graphing utility to approximate any points of intersection (accurate to three decimal places) of the graphs of equations. Verify your results algebraically.

75. $y = 9 - 2x$
$y = x - 3$

76. $y = \frac{1}{3}x + 2$
$y = \frac{5}{2}x - 11$

77. $y = 4 - x^2$
$y = 2x - 1$

78. $y = -x$
$y = 2x - x^2$

79. $y = 2x^2$
$y = x^4 - 2x^2$

80. $y = x^3 - 3$
$y = 5 - 2x$

In Exercises 81–88, solve the quadratic equation by factoring. Check your solutions in the original equation.

81. $6x^2 + 3x = 0$
82. $9x^2 - 1 = 0$
83. $x^2 - 2x - 8 = 0$
84. $x^2 - 10x + 9 = 0$
85. $3 + 5x - 2x^2 = 0$
86. $2x^2 = 19x + 33$
87. $x^2 + 4x = 12$
88. $-x^2 + 8x = 12$

In Exercises 89–96, solve the equation by extracting square roots. List both the exact solution and the decimal solution rounded to two decimal places.

89. $x^2 = 7$
90. $9x^2 = 25$
91. $(x - 12)^2 = 18$
92. $(x - 5)^2 = 20$
93. $(2x - 1)^2 = 18$
94. $(4x + 7)^2 = 44$
95. $(x - 7)^2 = (x + 3)^2$
96. $(x + 5)^2 = (x + 4)^2$

In Exercises 97–102, solve the quadratic equation by completing the square. Verify your answer graphically.

97. $x^2 + 4x - 32 = 0$
98. $x^2 - 2x - 3 = 0$
99. $x^2 + 6x + 2 = 0$
100. $x^2 + 8x + 14 = 0$
101. $9x^2 - 18x + 3 = 0$
102. $9x^2 - 12x - 14 = 0$

In Exercises 103–108, use the Quadratic Formula to solve the equation. Use a graphing utility to verify your solutions graphically.

103. $2 + 2x - x^2 = 0$
104. $x^2 - 10x + 22 = 0$
105. $x^2 + 8x - 4 = 0$
106. $4x^2 - 4x - 4 = 0$
107. $4x^2 + 16x + 15 = 0$
108. $9x^2 - 6x - 35 = 0$

In Exercises 109–114, solve the equation by any convenient method.

109. $x^2 - 2x - 1 = 0$
110. $11x^2 + 33x = 0$
111. $(x + 3)^2 = 81$
112. $x^2 - 14x + 49 = 0$
113. $x^2 - x - \frac{11}{4} = 0$
114. $x^2 + 3x - \frac{3}{4} = 0$

In Exercises 115–130, find all solutions of the equation. Use a graphing utility to verify the solutions graphically.

115. $4x^4 - 18x^2 = 0$
116. $20x^3 - 125x = 0$
117. $x^4 - 81 = 0$
118. $x^6 - 64 = 0$
119. $5x^3 + 30x^2 + 45x = 0$
120. $9x^4 - 24x^3 + 16x^2 = 0$
121. $x^3 - 3x^2 - x + 3 = 0$
122. $x^4 + 2x^3 - 8x - 16 = 0$
123. $x^4 - 4x^2 + 3 = 0$
124. $x^4 + 5x^2 - 36 = 0$
125. $4x^4 - 65x^2 + 16 = 0$
126. $36t^4 + 29t^2 - 7 = 0$
127. $\dfrac{1}{t^2} + \dfrac{8}{t} + 15 = 0$
128. $6\left(\dfrac{s}{s+1}\right)^2 + 5\left(\dfrac{s}{s+1}\right) - 6 = 0$
129. $2x + 9\sqrt{x} - 5 = 0$
130. $3x^{1/3} + 2x^{2/3} = 5$

In Exercises 131–150, find all solutions of the equation algebraically. Check your solutions both algebraically and graphically.

131. $\sqrt{x - 10} - 4 = 0$
132. $\sqrt[3]{2x + 5} + 3 = 0$
133. $\sqrt{x + 1} - 3x = 1$
134. $\sqrt{x + 5} = \sqrt{x - 5}$
135. $\sqrt{x} - \sqrt{x - 5} = 1$
136. $\sqrt{x} + \sqrt{x - 20} = 10$
137. $(x - 5)^{2/3} = 16$
138. $(x^2 - x - 22)^{4/3} = 16$
139. $3x(x - 1)^{1/2} + 2(x - 1)^{3/2} = 0$
140. $4x^2(x - 1)^{1/3} + 6x(x - 1)^{4/3} = 0$
141. $\dfrac{20 - x}{x} = x$
142. $\dfrac{4}{x} - \dfrac{5}{3} = \dfrac{x}{6}$
143. $\dfrac{1}{x} - \dfrac{1}{x + 1} = 3$
144. $\dfrac{x}{x^2 - 4} + \dfrac{1}{x + 2} = 3$
145. $x = \dfrac{3}{x} + \dfrac{1}{2}$
146. $4x + 1 = \dfrac{3}{x}$
147. $|2x - 1| = 5$
148. $|3x + 2| = 7$
149. $|x| = x^2 + x - 3$
150. $|x - 10| = x^2 - 10x$

Graphical Analysis **In Exercises 151–158, (a) use a graphing utility to graph the equation, (b) use the graph to approximate any x-intercepts of the graph, (c) set $y = 0$ and solve the resulting equation, and (d) compare the result with the x-intercepts of the graph.**

151. $y = x^3 - 2x^2 - 3x$ **152.** $y = x^4 - 10x^2 + 9$

153. $y = \sqrt{11x - 30} - x$ **154.** $y = 2x - \sqrt{15 - 4x}$

155. $y = \dfrac{1}{x} - \dfrac{4}{x - 1} - 1$ **156.** $y = x + \dfrac{9}{x + 1} - 5$

157. $y = |x + 1| - 2$ **158.** $y = |x - 2| - 3$

159. ***Per Capita Utilization*** The per capita utilization (in pounds) of nectarines and peaches N and cucumbers C from 1991 through 1996 can be modeled by

$$N = -0.37t + 6.88$$

$$C = 0.27t + 4.42$$

where $t = 1$ represents 1991. (Source: U.S. Department of Agriculture)

(a) What does the intersection of the graphs of these equations represent?

(b) Find the point of intersection of the graphs algebraically.

(c) Verify your answer to part (b) using the *zoom* and *trace* features of a graphing utility.

160. ***Saturated Steam*** The temperature T (in degrees Fahrenheit) of saturated steam increases as pressure increases. This relationship is approximated by

$$T = 75.82 - 2.11x + 43.51\sqrt{x}, \quad 5 \le x \le 40$$

where x is the absolute pressure in pounds per square inch.

(a) Use a graphing utility to graph the temperature equation over the specified domain.

(b) The temperature of steam at sea level ($x = 14.696$) is 212°F. Evaluate the model at this pressure and verify the result graphically.

(c) Use the model to approximate the pressure for a steam temperature of 240°F.

161. ***Demand Equation*** The marketing department at a publisher is asked to determine the price of a book. The department determines that the demand for the book depends on the price of the book according to the formula

$$p = 40 - \sqrt{0.0001x + 1}, \quad x \ge 0$$

where p is the price per book in dollars and x is the number of books sold at the given price. For instance, if the price were \$39, then (according to the model) no one would be willing to buy the book. On the other hand, if the price were \$17.60, 5 million copies could be sold.

(a) Use a graphing utility to graph the demand equation over the specified domain.

(b) If the publisher set the price at \$12.95, how many copies would be sold?

Synthesis

True or False? **In Exercises 162 and 163, determine whether the statement is true or false. Justify your answer.**

162. An equation can never have more than one extraneous solution.

163. Two linear equations can have either one point of intersection or no points of intersection.

164. ***Exploration*** Given that a and b are nonzero real numbers, determine the solutions of the equations.

(a) $ax^2 + bx = 0$ (b) $ax^2 - ax = 0$

P.5 Solving Inequalities Algebraically and Graphically

Properties of Inequalities

The inequality symbols $<, \leq, >,$ and $\geq$ are used to compare two numbers and to denote subsets of real numbers. For instance, the simple inequality $x \geq 3$ denotes all real numbers x that are greater than or equal to 3. In this section you will study inequalities that contain more involved statements such as

$$5x - 7 > 3x + 9 \qquad \text{and} \qquad -3 \leq 6x - 1 < 3.$$

As with an equation, you **solve an inequality** in the variable x by finding all values of x for which the inequality is true. These values are **solutions** of the inequality and are said to **satisfy** the inequality. For instance, the number 9 is a solution of the first inequality listed above because

$$5(9) - 7 > 3(9) + 9$$
$$38 > 36.$$

On the other hand, the number 7 is not a solution because

$$5(7) - 7 \not> 3(7) + 9$$
$$28 \not> 30.$$

The set of all real numbers that are solutions of an inequality is the **solution set** of the inequality.

The set of all points on the real number line that represent the solution set is the **graph of the inequality.** Graphs of many types of inequalities consist of intervals on the real number line.

The procedures for solving linear inequalities in one variable are much like those for solving linear equations. To isolate the variable you can make use of the **properties of inequalities.** These properties are similar to the properties of equality, but there are two important exceptions. When both sides of an inequality are multiplied or divided by a negative number, *the direction of the inequality symbol must be reversed.* Here is an example.

$-2 < 5$ Write original inequality.

$(-3)(-2) > (-3)(5)$ Multiply each side by -3 and reverse inequality.

$6 > -15$ New inequality

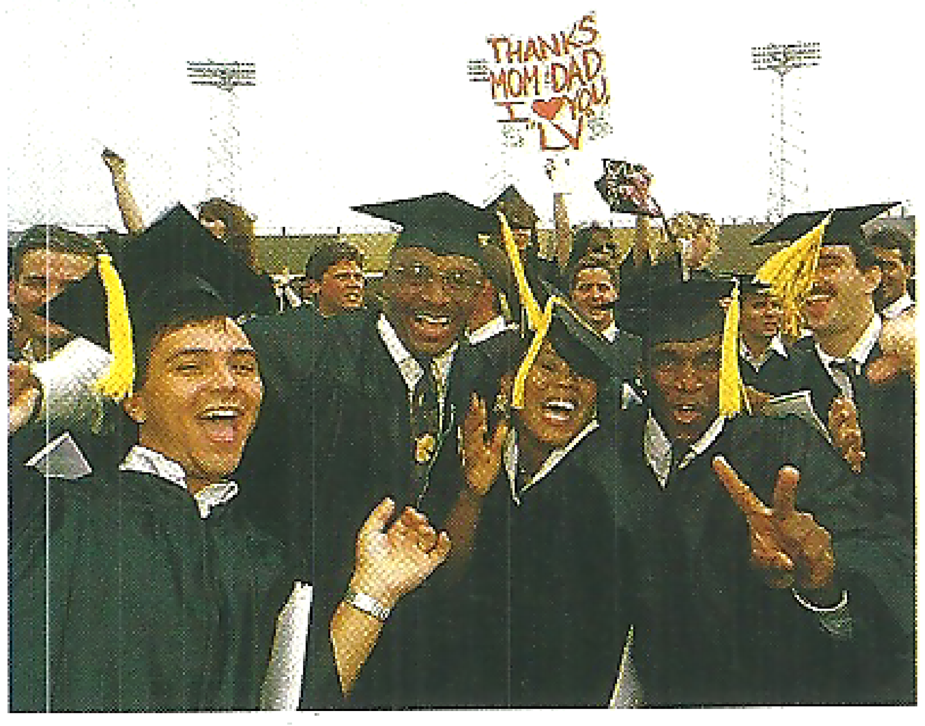
Two inequalities that have the same solution set are **equivalent inequalities.** The properties listed at the top of the next page describe operations that can be used to create equivalent inequalities.

What You Should Learn:

- How to use properties of inequalities to solve linear inequalities
- How to solve inequalities involving absolute values
- How to solve polynomial inequalities
- How to solve rational inequalities
- How to use inequalities to model and solve real-life problems

Why You Should Learn It:

An inequality can be used to describe the time when a level of real-life quantity is exceeded. For instance, Exercise 70 on page 64 shows how to use a quadratic inequality to determine when the total number of education degrees conferred in the United States exceeds 2.5 million.



Cliff Hollis/Liaison International

Properties of Inequalities

Let a, b, c, and d be real numbers.

1. *Transitive Property*

   $a < b$ and $b < c \quad \Rightarrow \quad a < c$

2. *Addition of Inequalities*

   $a < b$ and $c < d \quad \Rightarrow \quad a + c < b + d$

3. *Addition of a Constant*

   $a < b \quad \Rightarrow \quad a + c < b + c$

4. *Multiplying by a Constant*

   For $c > 0$, $a < b \quad \Rightarrow \quad ac < bc$

   For $c < 0$, $a < b \quad \Rightarrow \quad ac > bc$

Each of the properties above is true if the symbol $<$ is replaced by $\leq$ and $>$ is replaced by $\geq$. For instance, another form of Property 3 would be as follows.

$$a \leq b \quad \Rightarrow \quad a + c \leq b + c$$

Solving a Linear Inequality

The simplest type of inequality to solve is a **linear inequality** in a single variable, such as $2x + 3 > 4$. (See Appendix C for help with solving one-step linear inequalities.)

EXAMPLE 1 Solving a Linear Inequality

Solve the inequality

$$5x - 7 > 3x + 9.$$

Solution

$5x - 7 > 3x + 9$	Write original inequality.
$5x > 3x + 16$	Add 7 to each side.
$5x - 3x > 16$	Subtract $3x$ from each side.
$2x > 16$	Combine like terms.
$x > 8$	Divide each side by 2.

So, the solution set consists of all real numbers that are greater than 8. The interval notation for this solution set is $(8, \infty)$. The number line graph of this solution set is shown in Figure P.53.

Note that the five inequalities forming the solution steps of Example 1 are all *equivalent* in the sense that each has the same solution set.

Exploration

Use a graphing utility to graph $y_1 = 5x - 7$ and $y_2 = 3x + 9$ in the same viewing window. (Use $-1 \leq x \leq 15$ and $-5 \leq y \leq 50$.) For which values of x does the graph of y_1 lie above the graph of y_2? Explain how the answer to this question can be used to solve the inequality in Example 1.

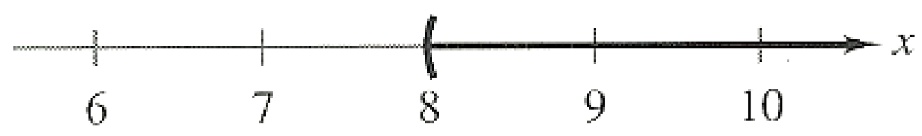


Figure P.53 *Solution interval:* $(8, \infty)$

STUDY TIP

Checking the solution set of an inequality is not as simple as checking the solution of an equation because there are simply too many x-values to substitute into the original inequality. However, you can get an indication of the validity of the solution set by substituting a few convenient values of x. For instance, in Example 1 try substituting $x = 5$ and $x = 10$ into the original inequality.

EXAMPLE 2 Solving an Inequality

Solve $1 - \frac{3x}{2} \geq x - 4$.

Algebraic Solution

$1 - \frac{3x}{2} \geq x - 4$ Write original inequality.

$2 - 3x \geq 2x - 8$ Multiply each side by the LCD.

$-3x \geq 2x - 10$ Subtract 2 from each side.

$-5x \geq -10$ Subtract $2x$ from each side.

$x \leq 2$ Divide each side by -5 and reverse inequality.

The solution set consists of all real numbers that are less than or equal to 2. The interval notation for this solution set is $(-\infty, 2]$. The number line graph of this solution set is shown in Figure P.54.

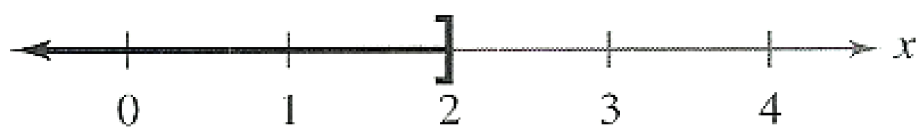


Figure P.54 *Solution interval:* $(-\infty, 2]$

Graphical Solution

Use a graphing utility to graph $y_1 = 1 - (3x)/2$ and $y_2 = x - 4$ in the same viewing window. In Figure P.55, you can see that the graphs appear to intersect at the point $(2, -2)$. Use the *intersect* feature of the graphing utility to confirm this. The graph of y_1 lies above the graph of y_2 to the left of their point of intersection, which implies that $y_1 \geq y_2$ for all $x \leq 2$.

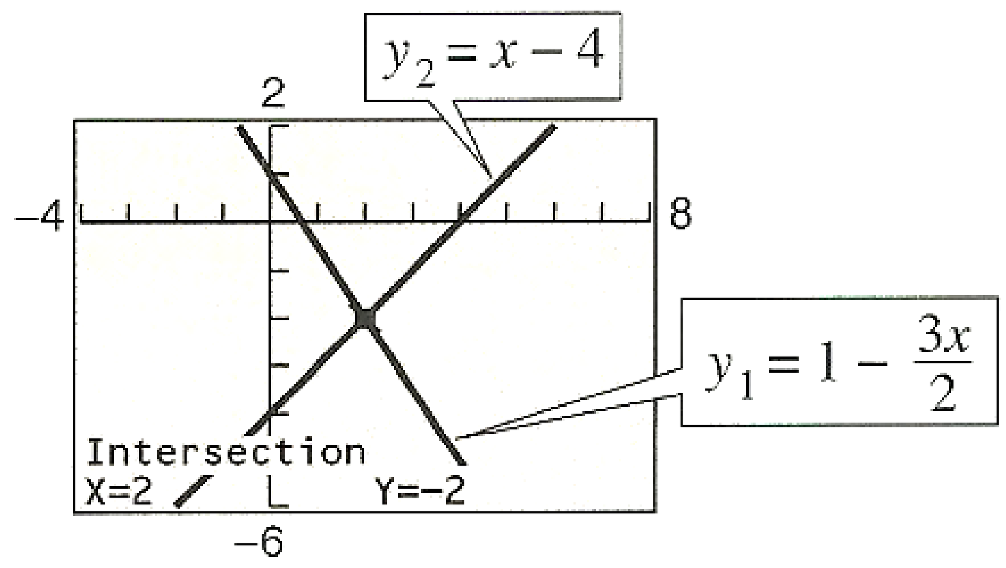


Figure P.55

Sometimes it is possible to write two inequalities as a **double inequality.** For instance, you can write the two inequalities $-4 \leq 5x - 2$ and $5x - 2 < 7$ more simply as $-4 \leq 5x - 2 < 7$, so you can solve the two inequalities together.

EXAMPLE 3 Solving a Double Inequality

Solve $-3 \leq 6x - 1 < 3$.

Algebraic Solution

$-3 \leq 6x - 1 < 3$ Write original inequality.

$-2 \leq 6x < 4$ Add 1 to all three parts.

$-\frac{1}{3} \leq x < \frac{2}{3}$ Divide by 6 and simplify.

The solution set consists of all real numbers that are greater than or equal to $-\frac{1}{3}$ *and* less than $\frac{2}{3}$. The interval notation for this solution set is $\left[-\frac{1}{3}, \frac{2}{3}\right)$. The number line graph of this solution set is shown in Figure P.56.

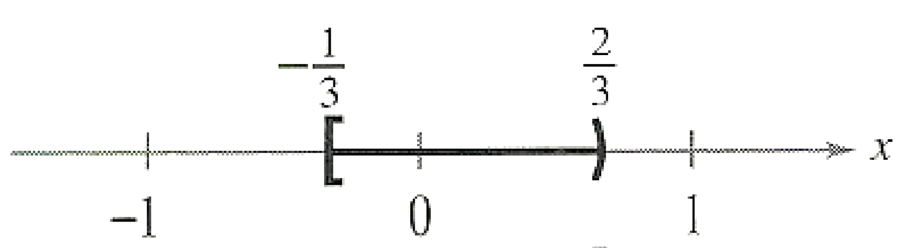


Figure P.56 *Solution interval:* $\left[-\frac{1}{3}, \frac{2}{3}\right)$

Graphical Solution

Use a graphing utility to graph $y_1 = 6x - 1$, $y_2 = -3$, and $y_3 = 3$ in the same viewing window. In Figure P.57, you can see that the graphs appear to intersect at the points $\left(-\frac{1}{3}, -3\right)$ and $\left(\frac{2}{3}, 3\right)$. Use the *intersect* feature of the graphing utility to confirm this. The graph of y_1 lies above the graph of y_2 to the right of $\left(-\frac{1}{3}, -3\right)$ *and* the graph of y_1 lies below the graph of y_3 to the left of $\left(\frac{2}{3}, 3\right)$. This implies that $y_2 \leq y_1 < y_3$ when $-\frac{1}{3} \leq x < \frac{2}{3}$.

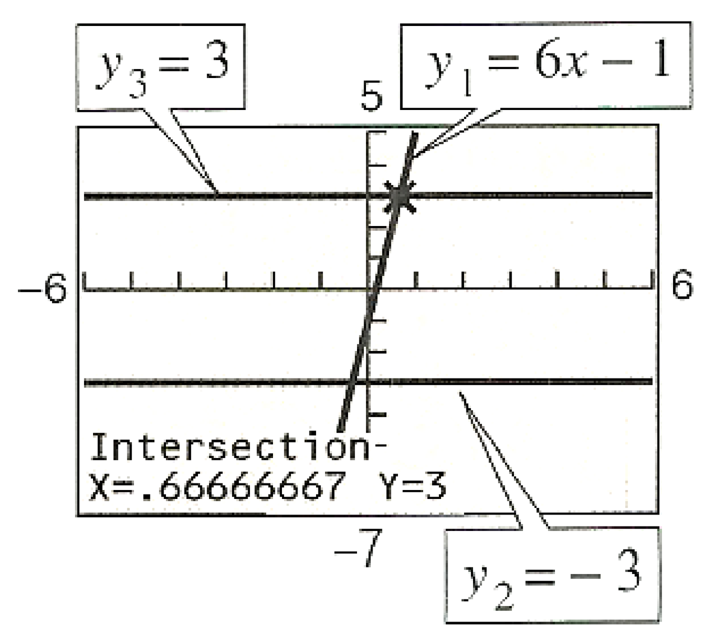


Figure P.57

Inequalities Involving Absolute Value

Solving an Absolute Value Inequality

Let x be a variable or an algebraic expression and let a be a real number such that $a \geq 0$.

1. The solutions of $|x| < a$ are all values of x that lie between $-a$ and a.

$$|x| < a \quad \text{if and only if} \quad -a < x < a.$$

2. The solutions of $|x| > a$ are all values of x that are less than $-a$ or greater than a.

$$|x| > a \quad \text{if and only if} \quad x < -a \quad \text{or} \quad x > a.$$

These rules are also valid if $<$ is replaced by $\leq$ and $>$ is replaced by $\geq$.

EXAMPLE 4 Solving Absolute Value Inequalities

Solve each inequality.

a. $|x - 5| < 2$ **b.** $|x - 5| > 2$

Algebraic Solution

a. $|x - 5| < 2$ Write original inequality.

$-2 < x - 5 < 2$ Equivalent inequalities

$-2 + 5 < x - 5 + 5 < 2 + 5$ Add 5 to all three parts.

$3 < x < 7$ Simplify.

The solution set is all real numbers that are greater than 3 *and* less than 7. The interval notation for this solution set is $(3, 7)$. The number line graph of this solution set is shown in Figure P.58.

b. The absolute value inequality $|x - 5| > 2$ is equivalent to the following compound inequality.

$$x - 5 < -2 \quad or \quad x - 5 > 2$$

Solve first inequality: $x - 5 < -2$ Write first inequality.

$x < 3$ Add 5 to each side.

Solve second inequality: $x - 5 > 2$ Write second inequality.

$x > 7$ Add 5 to each side.

The solution is all real numbers that are less than -3 *or* greater than 7. The interval notation for this solution set is $(-\infty, 3) \cup (7, \infty)$. The symbol $\cup$ is called a *union* symbol and is used to denote the combining of two sets. The number line graph of this solution set is shown in Figure P.59.

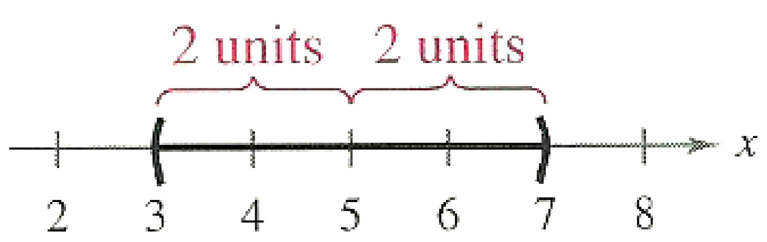


Figure P.58 $|x - 5| < 2$

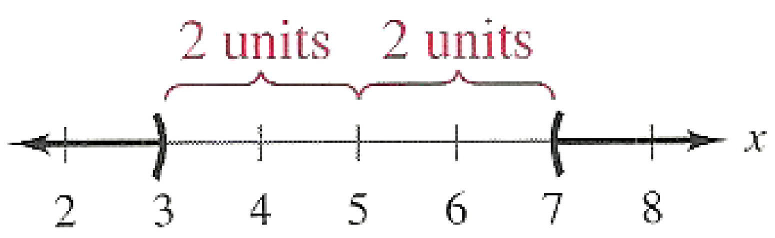


Figure P.59 $|x - 5| > 2$

Graphical Solution

a. Use a graphing utility to graph $y_1 = |x - 5|$ and $y_2 = 2$ in the same viewing window. In Figure P.60, you can see that the graphs appear to intersect at the points $(3, 2)$ and $(7, 2)$. Use the *intersect* feature of the graphing utility to confirm this. The graph of y_1 lies below the graph of y_2 when $3 < x < 7$. So, you can approximate the solution set to be all real numbers greater than 3 *and* less than 7.

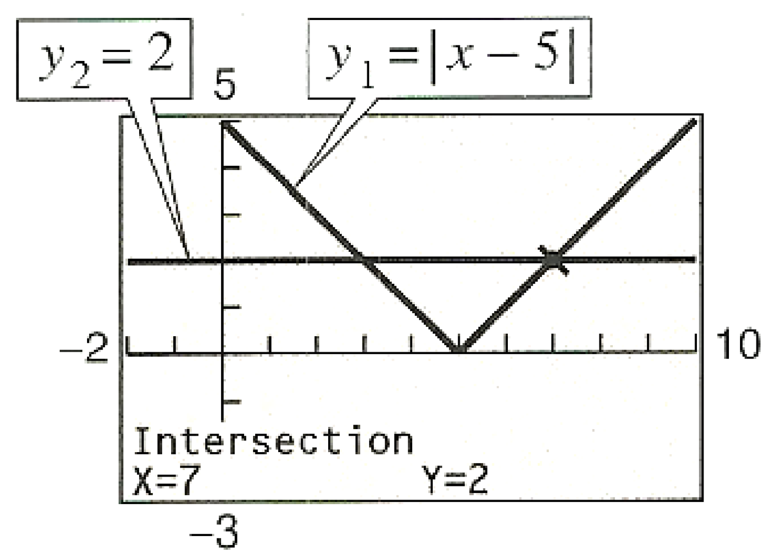


Figure P.60

b. In Figure P.60, you can see that the graph of y_1 lies above the graph of y_2 when $x < 3$ *or* when $x > 7$. So, you can approximate the solution set to be all real numbers that are less than 3 *or* greater than 7.

Polynomial Inequalities

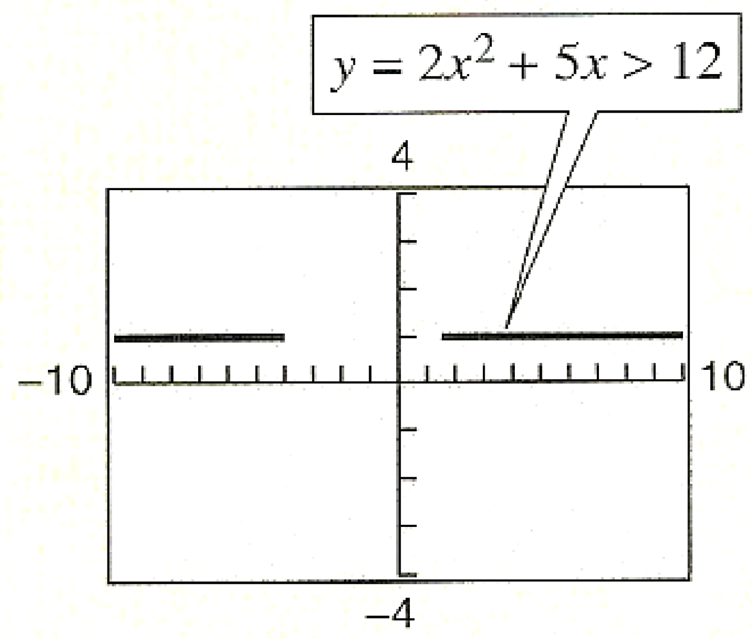
To solve a polynomial inequality such as $x^2 - 2x - 3 < 0$, use the fact that a polynomial can change signs only at its zeros (the x-values that make the polynomial equal to zero). Between two consecutive zeros a polynomial must be entirely positive or entirely negative. This means that when the real zeros of a polynomial are put in order, they divide the real number line into intervals in which the polynomial has no sign changes. These zeros are the **critical numbers** of the inequality, and the resulting open intervals are the **test intervals** for the inequality. For instance, the polynomial

$$x^2 - 2x - 3 = (x + 1)(x - 3)$$

has two zeros, $x = -1$ and $x = 3$, which divide the real number line into three test intervals: $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$. To solve the inequality $x^2 - 2x - 3 < 0$, you only need to test one value from each test interval.

STUDY TIP

Some graphing utilities will produce graphs of inequalities. For instance, you can graph $2x^2 + 5x > 12$ by setting the graphing utility to *dot* mode and entering $y = 2x^2 + 5x > 12$. Using $-10 \le x \le 10$ and $-4 \le y \le 4$, your graph should look like the graph shown below. Solve the problem algebraically to verify that the solution is $(-\infty, -4) \cup \left(\frac{3}{2}, \infty\right)$.



Finding Test Intervals for a Polynomial

To determine the intervals on which the values of a polynomial are entirely negative or entirely positive, use the following steps.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order. The zeros of a polynomial are its **critical numbers.**
2. Use the critical numbers of the polynomial to determine its **test intervals.**
3. Choose one representative x-value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, the polynomial will have negative values for *every* x-value in the interval. If the value of the polynomial is positive, the polynomial will have positive values for *every* x-value in the interval.

EXAMPLE 5 Investigating Polynomial Behavior

To determine the intervals on which $x^2 - x - 6$ is entirely negative and those on which it is entirely positive, factor the quadratic as

$$x^2 - x - 6 = (x + 2)(x - 3).$$

The critical numbers occur at $x = -2$ and $x = 3$. So, the test intervals for the quadratic are $(-\infty, -2)$, $(-2, 3)$, and $(3, \infty)$. In each test interval, choose a representative x-value and evaluate the polynomial, as shown in the table.

Interval	x-Value	Value of Polynomial	Sign of Polynomial
$(-\infty, -2)$	$x = -3$	$(-3)^2 - (-3) - 6 = 6$	Positive
$(-2, 3)$	$x = 0$	$(0)^2 - (0) - 6 = -6$	Negative
$(3, \infty)$	$x = 5$	$(5)^2 - (5) - 6 = 14$	Positive

The polynomial has negative values for every x in the interval $(-2, 3)$ and positive values for every x in the intervals $(-\infty, -2)$ and $(3, \infty)$. This result is shown graphically in Figure P.61.

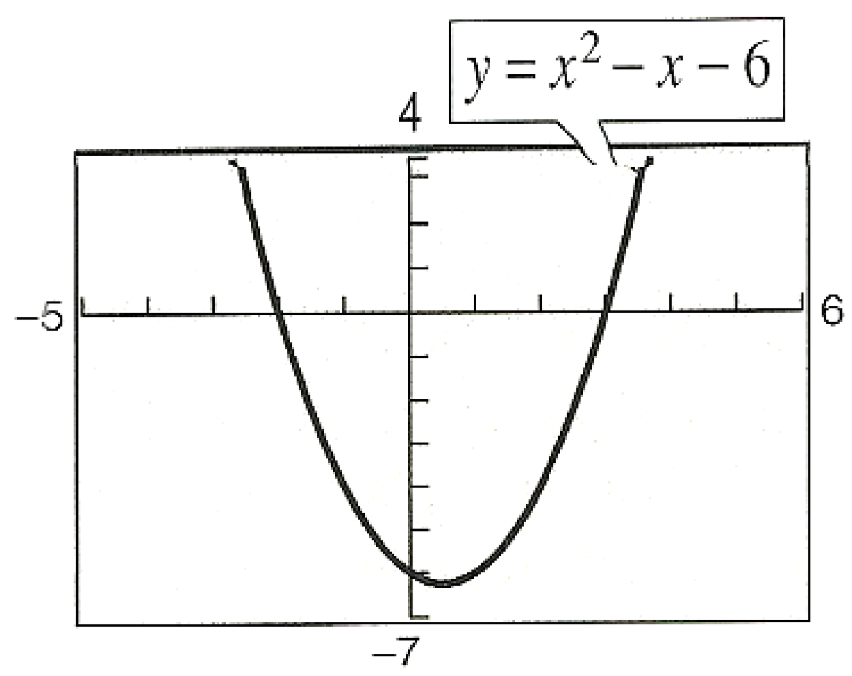


Figure P.61

To determine the test intervals for a polynomial inequality, the inequality must first be written in general form with the polynomial on one side.

EXAMPLE 6 Solving a Polynomial Inequality

Solve $2x^2 + 5x > 12$.

Algebraic Solution

$2x^2 + 5x > 12$ Write original inequality.

$2x^2 + 5x - 12 > 0$ Write in general form.

$(x + 4)(2x - 3) > 0$ Factor.

Critical Numbers: $x = -4, x = \frac{3}{2}$

Test Intervals: $(-\infty, -4), \left(-4, \frac{3}{2}\right), \left(\frac{3}{2}, \infty\right)$

Test: Is $(x + 4)(2x - 3) > 0$?

After testing these intervals, you can see that the polynomial $2x^2 + 5x - 12$ is positive in the open intervals $(-\infty, -4)$ and $\left(\frac{3}{2}, \infty\right)$. Therefore, the solution set of the inequality is

$$(-\infty, -4) \cup \left(\frac{3}{2}, \infty\right).$$

Graphical Solution

First write the polynomial inequality $2x^2 + 5x > 12$ as $2x^2 + 5x - 12 > 0$. Then use a graphing utility to graph $y = 2x^2 + 5x - 12$. In Figure P.62, you can see that the graph is *above* the x-axis when x is less than -4 *or* when x is greater than $\frac{3}{2}$. So, you can graphically approximate the solution set to be $(-\infty, -4) \cup \left(\frac{3}{2}, \infty\right)$.

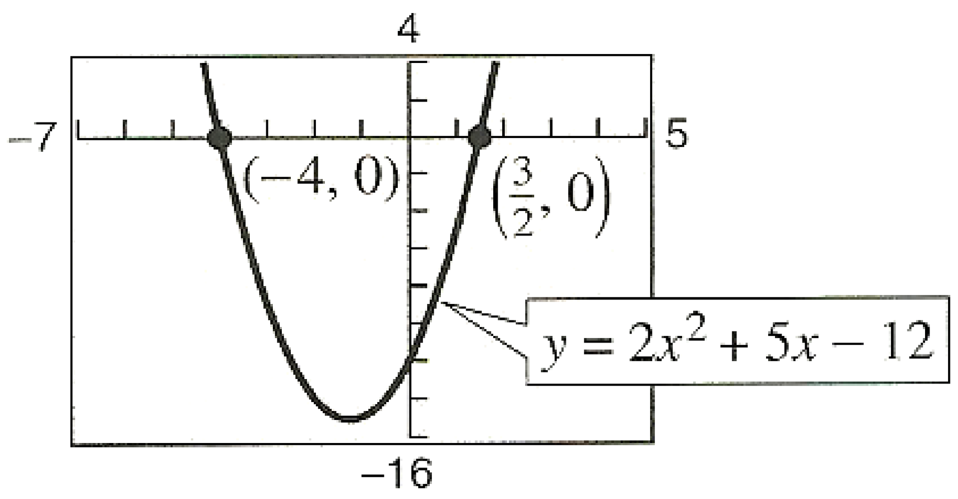


Figure P.62

EXAMPLE 7 Solving a Polynomial Inequality

Solve $2x^3 - 3x^2 - 32x > -48$.

Solution

Begin by writing the inequality in general form.

$2x^3 - 3x^2 - 32x > -48$ Write original inequality.

$2x^3 - 3x^2 - 32x + 48 > 0$ Write in general form.

$x^2(2x - 3) - 16(2x - 3) > 0$ Factor by grouping.

$(x^2 - 16)(2x - 3) > 0$ Distributive Property

$(x - 4)(x + 4)(2x - 3) > 0$ Factor difference of two squares.

The critical numbers are $x = -4$, $x = \frac{3}{2}$, and $x = 4$; and the test intervals are $(-\infty, -4)$, $\left(-4, \frac{3}{2}\right)$, $\left(\frac{3}{2}, 4\right)$, and $(4, \infty)$.

Interval	*x-Value*	*Polynomial Value*	*Conclusion*
$(-\infty, -4)$	$x = -5$	$2(-5)^3 - 3(-5)^2 - 32(-5) + 48 = -117$	Negative
$\left(-4, \frac{3}{2}\right)$	$x = 0$	$2(0)^3 - 3(0)^2 - 32(0) + 48 = 48$	Positive
$\left(\frac{3}{2}, 4\right)$	$x = 2$	$2(2)^3 - 3(2)^2 - 32(2) + 48 = -12$	Negative
$(4, \infty)$	$x = 5$	$2(5)^3 - 3(5)^2 - 32(5) + 48 = 63$	Positive

From this you can conclude that the polynomial $2x^3 - 3x^2 - 32x + 48$ is positive on the open intervals $\left(-4, \frac{3}{2}\right)$ and $(4, \infty)$. Therefore, the solution set consists of all real numbers in the intervals $\left(-4, \frac{3}{2}\right)$ and $(4, \infty)$.

STUDY TIP

When solving a quadratic inequality, be sure you have accounted for the particular type of inequality symbol given in the inequality. For instance, in Example 7, note that the original inequality contained a "greater than" symbol and the solution consisted of two open intervals. If the original inequality had been

$$2x^3 - 3x^2 + 32x \geq -48,$$

the solution would have consisted of the closed interval $\left[-4, \frac{3}{2}\right]$ and the interval $[4, \infty)$.

EXAMPLE 8 Unusual Solution Sets

a. The solution set of

$$x^2 + 2x + 4 > 0$$

consists of the entire set of real numbers, $(-\infty, \infty)$. In other words, the quadratic $x^2 + 2x + 4$ is positive for every real value of x, as indicated in Figure P.63(a). (Note that this quadratic inequality has *no* critical numbers. In such a case, there is only one test interval—the entire real number line.)

b. The solution set of

$$x^2 + 2x + 1 \leq 0$$

consists of the single real number -1, because the graph touches the x-axis only at -1, as shown in Figure P.63(b).

c. The solution set of

$$x^2 + 3x + 5 < 0$$

is empty. In other words, the quadratic $x^2 + 3x + 5$ is not less than zero for any value of x, as indicated in Figure P.63(c).

d. The solution set of

$$x^2 - 4x + 4 > 0$$

consists of all real numbers *except* the number 2. In interval notation, this solution set can be written as $(-\infty, 2) \cup (2, \infty)$. The graph of $x^2 - 4x + 4$ lies above the x-axis except at $x = 2$, where it touches it, as indicated in Figure P.63(d).

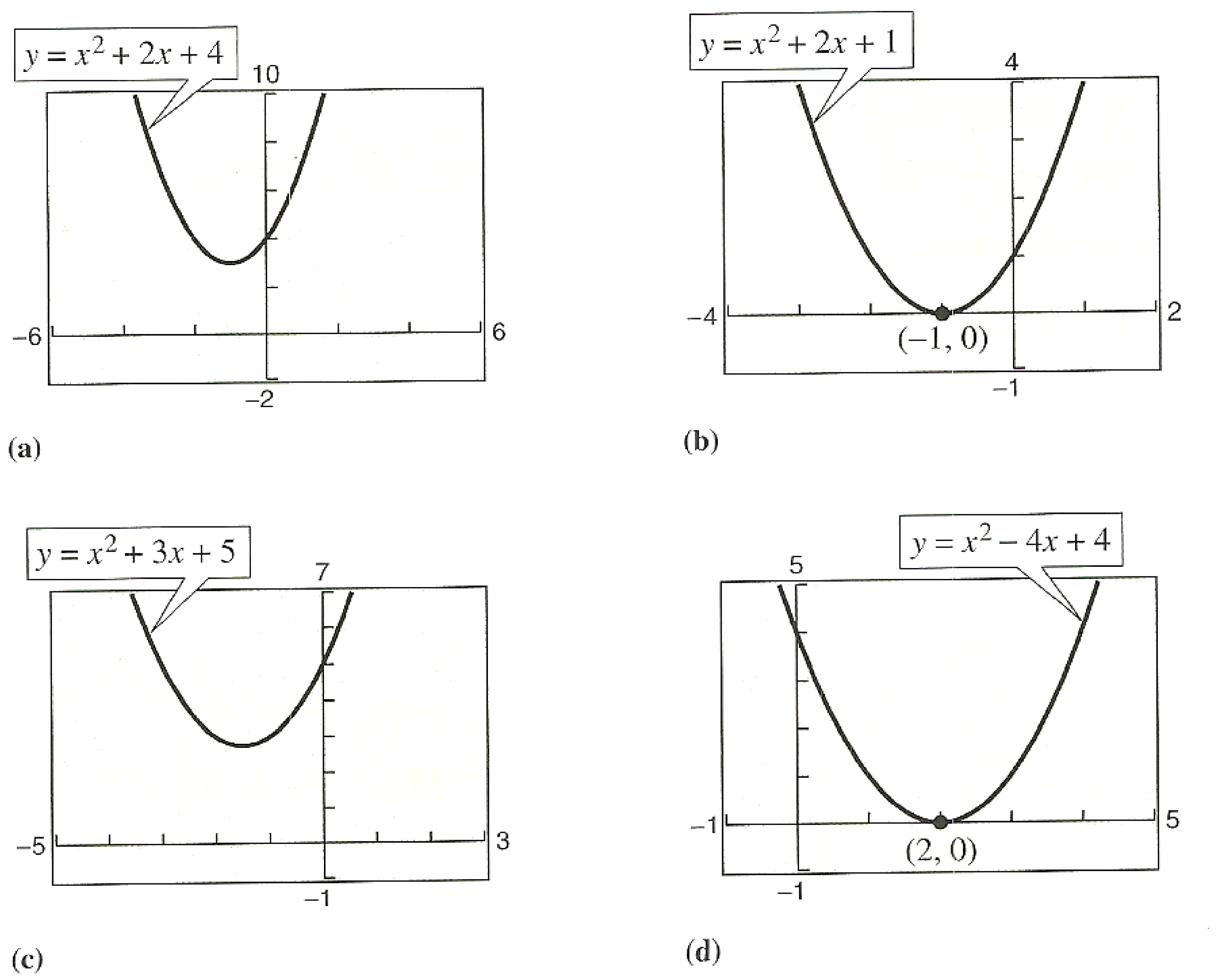


Figure P.63

STUDY TIP

One of the advantages of technology is that you can solve complicated polynomial inequalities that might be difficult, or even impossible, to factor. For instance, you could use a graphing utility to approximate the solution to the inequality

$$x^3 - 0.26x^2 - 3.1416x + 1.414 < 0.$$

Rational Inequalities

The concepts of critical numbers and test intervals can be extended to inequalities involving rational expressions. To do this, use the fact that the value of a rational expression can change sign only at its *zeros* (the x-values for which its numerator is zero) and its *undefined values* (the x-values for which its denominator is zero). These two types of numbers make up the *critical numbers* of a rational inequality.

EXAMPLE 9 Solving a Rational Inequality

Solve $\dfrac{2x-7}{x-5} \le 3$.

Algebraic Solution

$$\frac{2x-7}{x-5} \le 3 \qquad \text{Write original inequality.}$$

$$\frac{2x-7}{x-5} - 3 \le 0 \qquad \text{Write in standard form.}$$

$$\frac{2x-7-3x+15}{x-5} \le 0 \qquad \text{Write as single fraction.}$$

$$\frac{-x+8}{x-5} \le 0 \qquad \text{Simplify.}$$

Now, in standard form you can see that the critical numbers are 5 and 8, and you can proceed as follows.

Critical Numbers: $x = 5, x = 8$

Test Intervals: $(-\infty, 5), (5, 8), (8, \infty)$

Test: Is $\dfrac{-x+8}{x-5} \le 0$?

Interval	*x-Value*	*Polynomial Value*	*Conclusion*
$(-\infty, 5)$	$x = 0$	$\frac{-0+8}{0-5} = -\frac{8}{5}$	Negative
$(5, 8)$	$x = 6$	$\frac{-6+8}{6-5} = 2$	Positive
$(8, \infty)$	$x = 9$	$\frac{-9+8}{9-5} = -\frac{1}{4}$	Negative

By testing these intervals, you can determine that the rational expression $(-x+8)/(x-5)$ is negative in the open intervals $(-\infty, 5)$ and $(8, \infty)$. Moreover, because $(-x+8)/(x-5) = 0$ when $x = 8$, you can conclude that the solution set of the inequality is $(-\infty, 5) \cup [8, \infty)$.

Graphical Solution

Use a graphing utility to graph

$$y_1 = \frac{2x-7}{x-5} \text{ and } y_2 = 3$$

in the same viewing window. In Figure P.64, you can see that the graphs appear to intersect at the point $(8, 3)$. Use the *intersect* feature of the graphing utility to confirm this. The graph of y_1 lies below the graph of y_2 in the intervals $(-\infty, 5)$ and $[8, \infty)$. So, you can graphically approximate the solution set to be all real numbers less than 5 *or* all real numbers greater than or equal to 8.

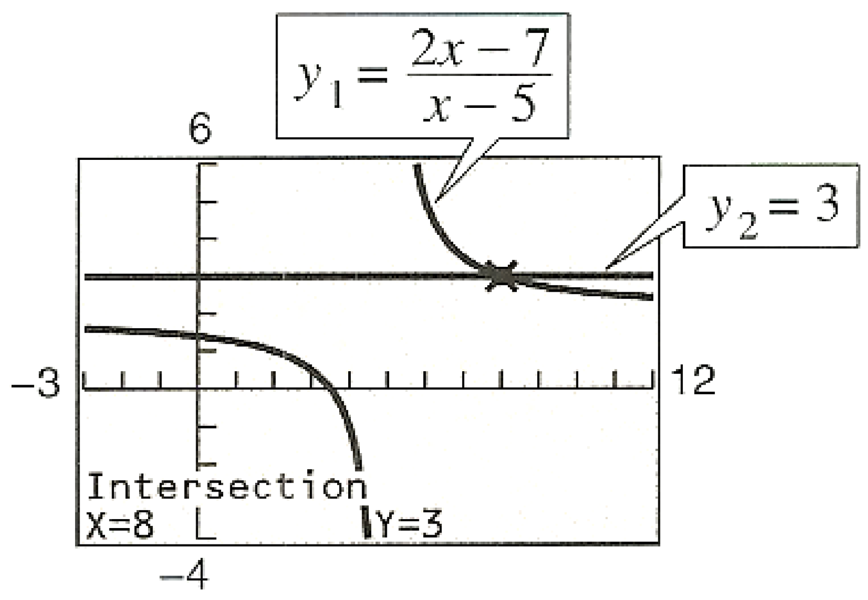


Figure P.64

Note in Example 9 that $x = 5$ is not included in the solution set because the inequality is undefined when $x = 5$.

Applications

The *implied domain* of a function is the set of all x-values for which a function is defined. A common type of implied domain is that used to avoid even roots of negative numbers, as shown in Example 10.

EXAMPLE 10 Finding the Domain of an Expression

Find the domain of

$$\sqrt{64 - 4x^2}.$$

Solution

Because $\sqrt{64 - 4x^2}$ is defined only if $64 - 4x^2$ is nonnegative, the domain is given by $64 - 4x^2 \geq 0$.

$$64 - 4x^2 \geq 0 \qquad \text{Write in general form.}$$

$$16 - x^2 \geq 0 \qquad \text{Divide each side by 4.}$$

$$(4 - x)(4 + x) \geq 0 \qquad \text{Factor.}$$

The inequality has two critical numbers: -4 and 4. A test shows that $64 - 4x^2 \geq 0$ in the *closed interval* $[-4, 4]$. The graph of $y = \sqrt{64 - 4x^2}$, shown in Figure P.65, confirms that the domain is $[-4, 4]$.

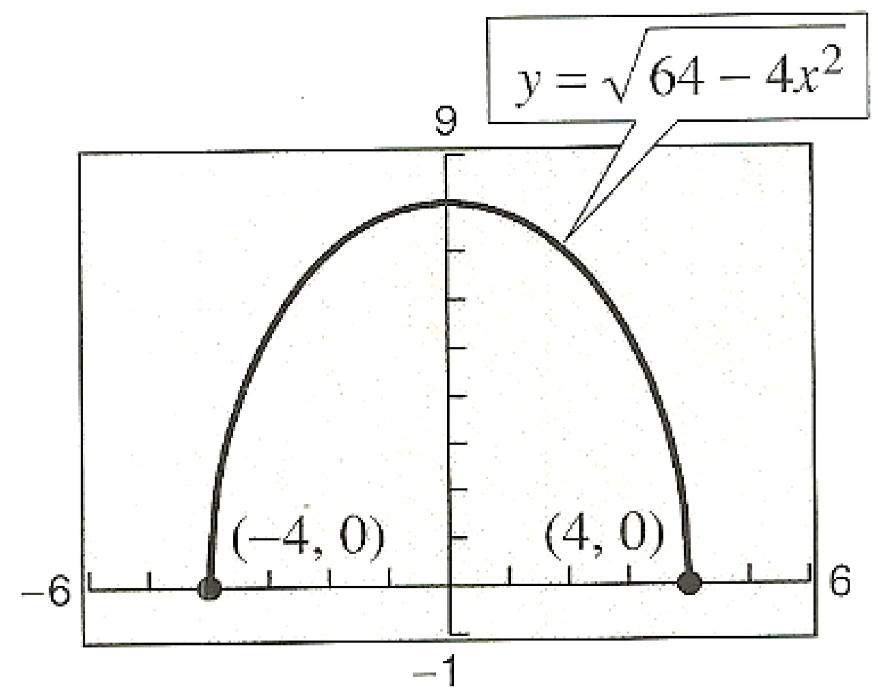


Figure P.65

EXAMPLE 11 The Height of a Projectile

A projectile is fired straight upward from ground level with an initial velocity of 384 feet per second. During what time period will its height exceed 2000 feet?

Solution

The position of an object moving vertically can be modeled by the *position equation*

$$s = -16t^2 + v_0t + s_0,$$

where s is the height in feet and t is the time in seconds. In this case, $s_0 = 0$ and $v_0 = 384$. So, you need to solve the inequality $-16t^2 + 384t > 2000$. Using a graphing utility, graph $s = -16t^2 + 384t$ and $s = 2000$, as shown in Figure P.66. From the graph, you can determine that $-16t^2 + 384t > 2000$ for t between approximately 7.6 and 16.4. You can verify this result algebraically as follows.

$$-16t^2 + 384t > 2000 \qquad \text{Write original inequality.}$$

$$t^2 - 24t < -125 \qquad \text{Divide by } -16 \text{ and reverse inequality.}$$

$$t^2 - 24t + 125 < 0 \qquad \text{Write in general form.}$$

By the Quadratic Formula the critical numbers are $12 - \sqrt{19}$ and $12 + \sqrt{19}$, or approximately 7.64 and 16.36. A test will verify that the height of the projectile will exceed 2000 feet when $7.64 < t < 16.36$, that is, during the time interval (7.64, 16.36) seconds.

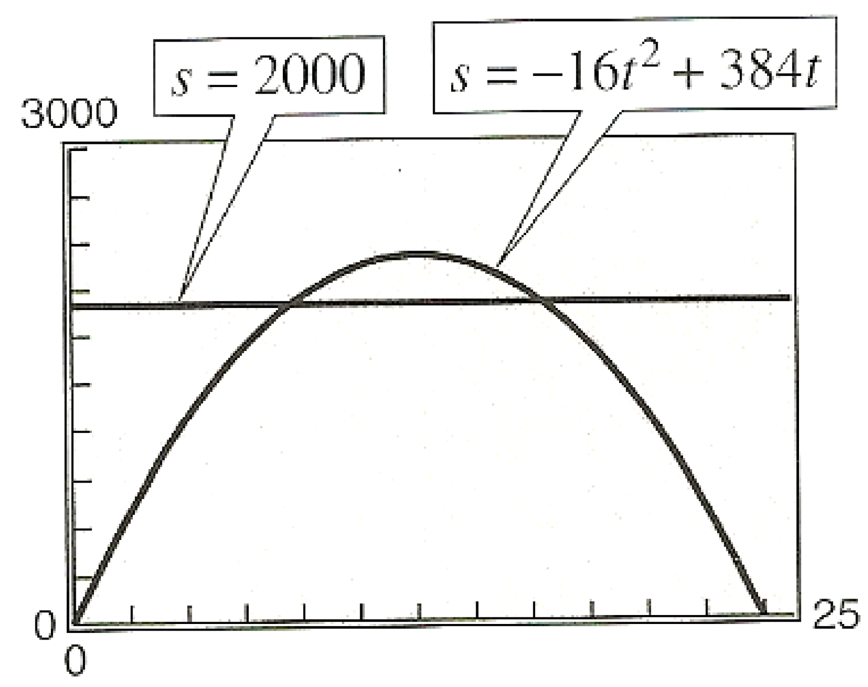
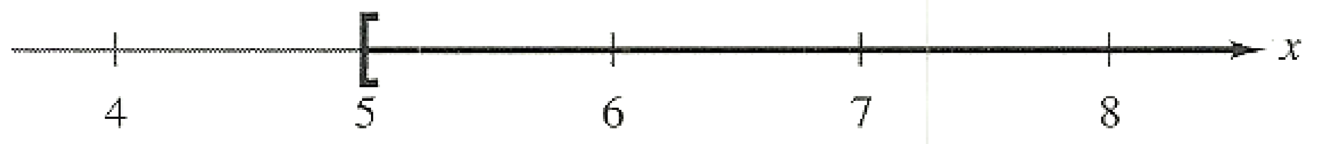
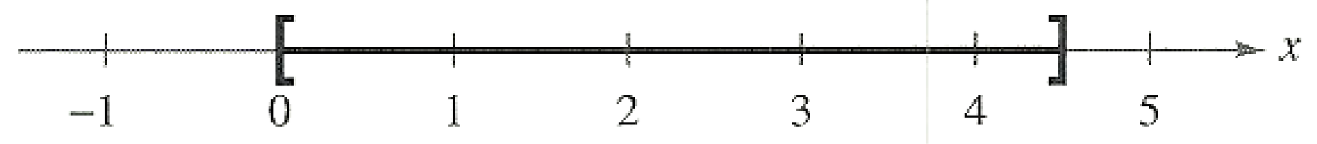


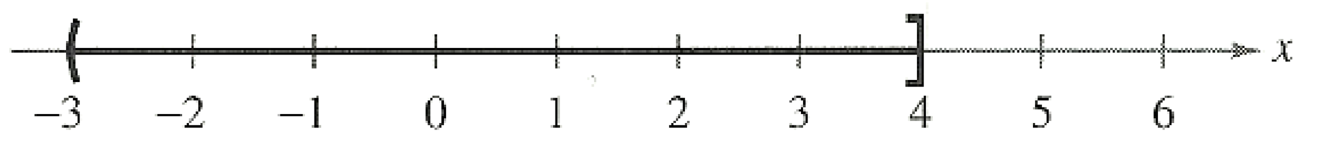
Figure P.66

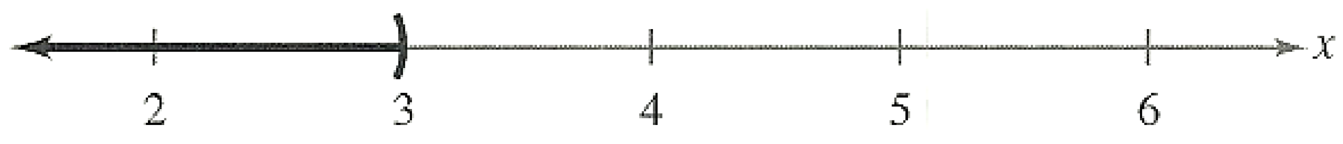
P.5 Exercises

In Exercises 1–4, match the inequality with its graph. [The graphs are labeled (a), (b), (c), and (d).]

(a) 

(b) 

(c) 

(d) 

1. $x < 3$ **2.** $x \geq 5$

3. $-3 < x \leq 4$ **4.** $0 \leq x \leq \frac{9}{2}$

In Exercises 5–8, determine whether each given value of x is a solution of the inequality.

Inequality	*Values*	
5. $5x - 12 > 0$	(a) $x = 3$	(b) $x = -3$
	(c) $x = \frac{5}{2}$	(d) $x = \frac{3}{2}$
6. $-5 < 2x - 1 \leq 1$	(a) $x = -\frac{1}{2}$	(b) $x = -\frac{5}{2}$
	(c) $x = \frac{4}{3}$	(d) $x = 0$
7. $-1 < \frac{3 - x}{2} \leq 1$	(a) $x = 0$	(b) $x = \sqrt{5}$
	(c) $x = 1$	(d) $x = 5$
8. $\lvert x - 10 \rvert \geq 3$	(a) $x = 13$	(b) $x = -1$
	(c) $x = 14$	(d) $x = 9$

In Exercises 9–18, solve the inequality and sketch the solution on the real number line. Use a graphing utility to verify your solution graphically.

9. $-10x < 40$ **10.** $2x > 3$

11. $4(x + 1) < 2x + 3$ **12.** $2x + 7 < 3$

13. $1 < 2x + 3 < 9$ **14.** $-2 < 3x + 1 < 10$

15. $-8 \leq 1 - 3(x - 2) < 13$

16. $0 \leq 2(x + 4) < 20$

17. $-4 < \frac{2x - 3}{3} < 4$ **18.** $0 \leq \frac{x + 3}{2} < 5$

Graphical Analysis **In Exercises 19–24, use a graphing utility to approximate the solution.**

19. $6x > 12$ **20.** $3x - 1 \leq 5$

21. $5 - 2x \geq 1$ **22.** $3(x + 1) < x + 7$

23. $-9 < 6x - 1 < 1$ **24.** $-10 < 4(x - 3) \leq 8$

In Exercises 25–28, use a graphing utility to graph the equation and graphically approximate the values of x that satisfy the specified inequalities. Then solve each inequality algebraically.

Equation	*Inequalities*	
25. $y = 2x - 3$	(a) $y \geq 1$	(b) $y \leq 0$
26. $y = \frac{2}{3}x + 1$	(a) $y \leq 5$	(b) $y \geq 0$
27. $y = -\frac{1}{2}x + 2$	(a) $0 \leq y \leq 3$	(b) $y \geq 0$
28. $y = -3x + 8$	(a) $-1 \leq y \leq 3$	(b) $y \leq 0$

In Exercises 29–36, solve the inequality and sketch the solution on the real number line.

29. $\lvert 5x \rvert > 10$ **30.** $\lvert x - 20 \rvert \leq 4$

31. $\lvert x - 7 \rvert < 6$ **32.** $\lvert x - 20 \rvert \geq 4$

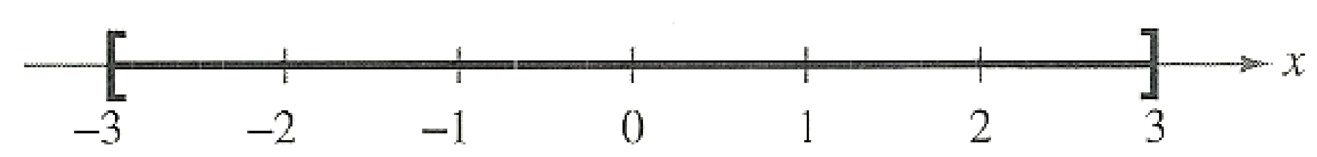
33. $\lvert x + 14 \rvert + 3 > 17$ **34.** $\left\lvert \frac{x - 3}{2} \right\rvert \geq 5$

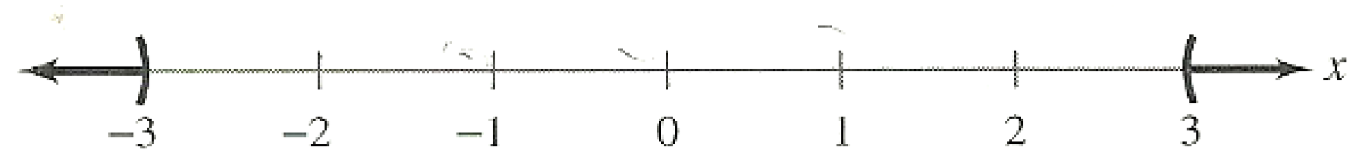
35. $\lvert 1 - 2x \rvert < 5$ **36.** $3\lvert 4 - 5x \rvert \leq 9$

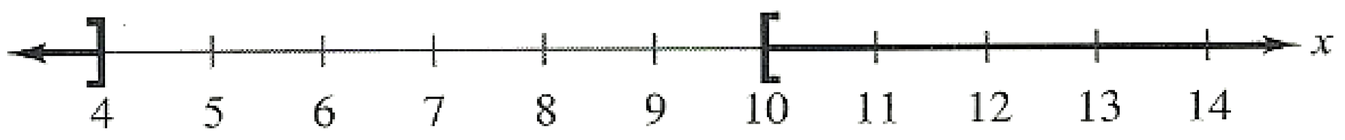
In Exercises 37 and 38, use a graphing utility to graph the equation and graphically approximate the values of x that satisfy the specified inequalities. Then solve each inequality algebraically.

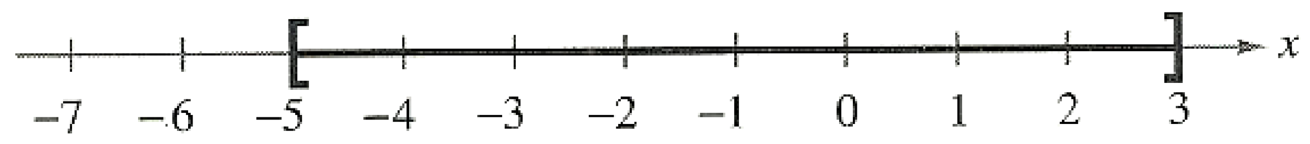
Equation	*Inequalities*	
37. $y = \lvert x - 3 \rvert$	(a) $y \leq 2$	(b) $y \geq 4$
38. $y = \left\lvert \frac{1}{2}x + 1 \right\rvert$	(a) $y \leq 4$	(b) $y \geq 1$

In Exercises 39–44, use absolute value notation to define each interval (or pair of intervals) on the real number line.

39. 

40. 

41. 

42. 

43. All real numbers within 10 units of 12

44. All real numbers whose distances from -3 are more than 5

In Exercises 45–50, solve the inequality and graph the solution on the real number line. Use a graphing utility to verify your solution graphically.

45. $(x + 2)^2 < 25$

46. $(x + 6)^2 \leq 8$

47. $x^2 + 4x + 4 \geq 9$

48. $x^2 - 6x + 9 < 16$

49. $x^3 - 4x \geq 0$

50. $x^4(x - 3) \leq 0$

In Exercises 51–54, use a graphing utility to graph the equation and graphically approximate the values of x that satisfy the specified inequalities. Then solve each inequality algebraically.

	Equation	*Inequalities*
51.	$y = -x^2 + 2x + 3$	(a) $y \leq 0$ (b) $y \geq 3$
52.	$y = \frac{1}{2}x^2 - 2x + 1$	(a) $y \leq 1$ (b) $y \geq 7$
53.	$y = \frac{1}{8}x^3 - \frac{1}{2}x$	(a) $y \geq 0$ (b) $y \leq 6$
54.	$y = x^3 - x^2 - 16x + 16$	(a) $y \leq 0$ (b) $y \geq 36$

In Exercises 55–58, solve the inequality and graph the solution on the real number line. Use a graphing utility to verify your solution graphically.

55. $\dfrac{1}{x} - x > 0$

56. $\dfrac{1}{x} - 4 < 0$

57. $\dfrac{x + 6}{x + 1} - 2 < 0$

58. $\dfrac{x + 12}{x + 2} - 3 \geq 0$

In Exercises 59–62, use a graphing utility to graph the equation and graphically approximate the values of x that satisfy the specified inequalities. Then solve each inequality algebraically.

	Equation	*Inequalities*	
59.	$y = \dfrac{3x}{x - 2}$	(a) $y \leq 0$	(b) $y \geq 6$
60.	$y = \dfrac{2(x - 2)}{x + 1}$	(a) $y \leq 0$	(b) $y \geq 8$
61.	$y = \dfrac{2x^2}{x^2 + 4}$	(a) $y \geq 1$	(b) $y \leq 2$
62.	$y = \dfrac{5x}{x^2 + 4}$	(a) $y \geq 1$	(b) $y \leq 0$

In Exercises 63–68, find the domain of x in the expression.

63. $\sqrt{x - 5}$

64. $\sqrt{x^2 - 4}$

65. $\sqrt[3]{6 - x}$

66. $\sqrt[3]{2x^2 - 8}$

67. $\sqrt[4]{6x + 15}$

68. $\sqrt[4]{4 - x^2}$

69. ***Data Analysis*** You want to determine whether there is a relationship between an athlete's weight x (in pounds) and the athlete's maximum bench-press weight y (in pounds). The table shows a sample of 12 athletes.

x	165	184	150	210	196	240
y	170	185	200	255	205	295

x	202	170	185	190	230	160
y	190	175	195	185	250	155

(a) Use a graphing utility to plot the data.

(b) A model for this data is $y = 1.266x - 35.766$. Use a graphing utility to graph the equation on the same display used in part (a).

(c) Use the graph to estimate the values of x that estimate a maximum bench-press weight of at least 200 pounds.

(d) Use the graph to write a statement about the accuracy of the model. If you think the graph indicates that an athlete's weight is not a particularly good indicator of the athlete's maximum bench-press weight, list other factors that may influence an individual's maximum bench-press weight.

70. ***Educational Degrees*** The number D (in thousands) of earned degrees conferred annually in the United States from 1950 to 1995 is approximated by the model

$$D = -0.0977t^2 + 47.1174t + 291.5651$$

where $t = 0$ represents 1950. (Source: U.S. National Center for Education Statistics)

(a) Use a graphing utility to graph the model.

(b) According to this model, estimate when the number of degrees will exceed 2,500,000.

71. ***Height*** The height h of two-thirds of the members of a certain population satisfies the inequality

$$\left|\frac{h-68.5}{2.7}\right| \le 1$$

where h is measured in inches. Determine the interval on the real number line in which these heights lie.

72. ***Meteorology*** A certain electronic device is to be operated in an environment with relative humidity h in the interval defined by

$$|h - 50| \le 30.$$

What are the minimum and maximum relative humidities for the operation of this device?

73. ***Music*** Michael Kasha of Florida State University used physics and mathematics to design a new classical guitar. He used the model for the frequency of the vibrations on a circular plate

$$v = \frac{2.6t}{d^2}\sqrt{\frac{E}{\rho}}$$

where v is the frequency, t is the plate thickness, d is the diameter, E is the elasticity of the plate material, and ρ is the density of the plate material. For fixed values of d, E, and ρ, the graph of the equation is a line, as shown below.

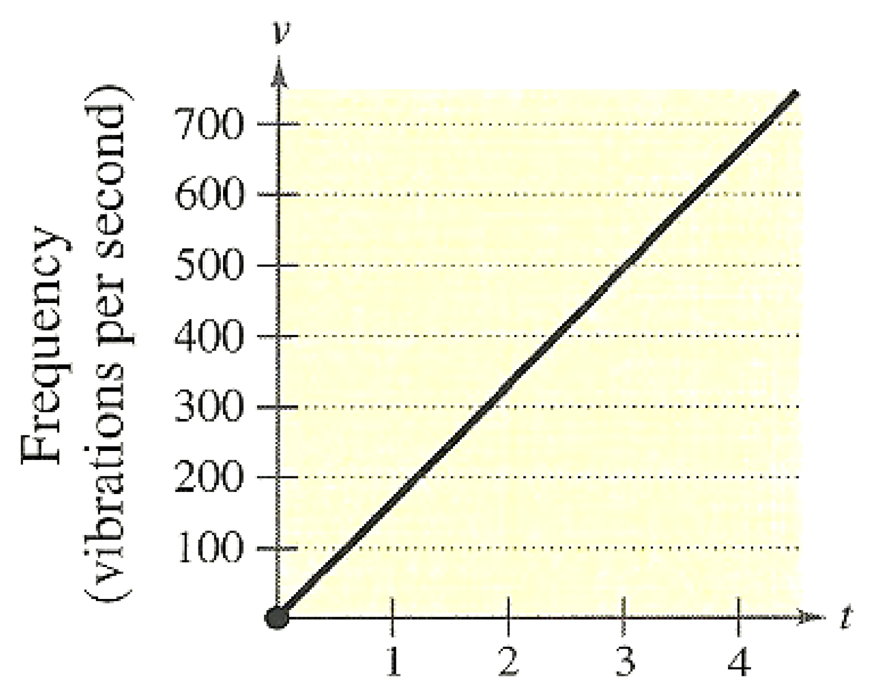


Plate thickness (millimeters)

(a) Estimate the frequency if the plate thickness is 2 millimeters.

(b) Estimate the plate thickness if the frequency is 600 vibrations per second.

(c) Approximate the interval for the plate thickness if the frequency is between 200 and 400 vibrations per second.

(d) Approximate the interval for the frequency if the plate thickness is less than 3 millimeters.

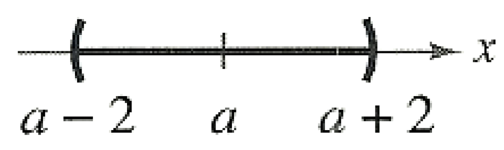
Synthesis

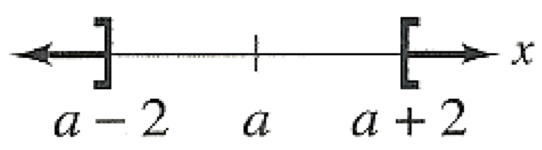
True or False? **In Exercises 74 and 75, determine whether the statement is true or false. Justify your answer.**

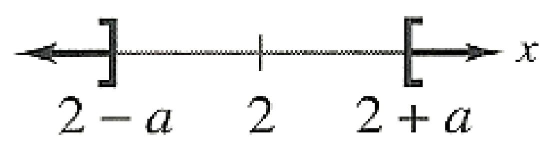
74. If a, b, and c are real numbers, and $a \le b$, then $ac \le bc$.

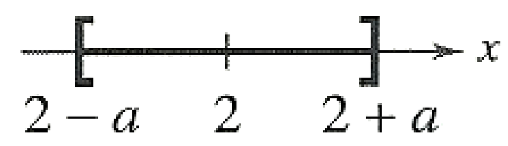
75. If $-10 \le x \le 8$, then $-10 \ge -x$ and $-x \ge -8$.

76. Identify the solution of the inequality $|x - a| \ge 2$.

(a) 

(b) 

(c) 

(d) 

77. Consider the polynomial $(x - a)(x - b)$ and the real number line.

(a) Identify the points on the line where the polynomial is zero.

(b) In each of the three subintervals of the line, write the sign of each factor and the sign of the product.

(c) For which x-values does a polynomial possibly change signs?

P Chapter Summary

What did you learn?

	Review Exercises
Section P.1	
☐ How to plot points in the Cartesian plane	1–6
☐ How to represent data graphically using scatter plots, bar graphs, and line graphs	7, 8
☐ How to use the Distance Formula to find the distance between two points	9–12
☐ How to use the Midpoint Formula to find the midpoint of a line segment	13–15
☐ How to find the equation of a circle	16, 17
Section P.2	
☐ How to sketch graphs of equations by point plotting	18–29
☐ How to sketch graphs of equations using a graphing utility	30–37
☐ How to use graphs of equations in real-life problems	38
Section P.3	
☐ How to find slopes of lines	39–48
☐ How to write linear equations given points on lines and their slopes	49–58
☐ How to use slope-intercept forms of linear equations to sketch graphs of lines	59–64
☐ How to use slope to identify parallel and perpendicular lines	65–68
Section P.4	
☐ How to solve linear equations	69–72
☐ How to find x- and y-intercepts of graphs of equations	73–78
☐ How to find solutions of equations graphically	79–84
☐ How to find the points of intersection of two graphs	85–88
☐ How to solve polynomial equations	89–102
☐ How to solve equations involving radicals, fractions, or absolute values	103–116
Section P.5	
☐ How to use properties of inequalities to solve linear inequalities	117–120
☐ How to solve inequalities involving absolute values	121–126
☐ How to solve polynomial inequalities	127–130
☐ How to solve rational inequalities	131–134
☐ How to use inequalities to model and solve real-life problems	135

P Review Exercises

P.1 In Exercises 1–4, plot the point and determine the quadrant in which it is located.

1. $(8, -3)$
2. $(-4, -9)$
3. $\left(-\frac{5}{2}, 10\right)$
4. $(-6.5, -0.5)$

In Exercises 5 and 6, determine the quadrant(s) in which (x, y) is located so that the conditions are satisfied.

5. $x > 0$ and $y = -2$
6. (x, y), $xy = 4$

7. ***Patents*** The number of patents P (in thousands) issued in the United States from 1988 through 1996 is shown in the table. (Source: U.S. Patent and Trademark Office)

Year	1988	1989	1990	1991	1992
P	84.4	102.7	99.2	106.8	107.4

Year	1993	1994	1995	1996
P	109.7	113.6	113.8	121.7

(a) Sketch a scatter plot of the data.

(b) What statement can be made about the number of patents issued in the United States?

8. ***Business*** The net profits (in millions of dollars) for the Progressive Corporation for the years 1994 through 1998 are shown in the table. Create a bar graph and a line graph for the data. (Source: Progressive Corporation)

Year	1994	1995	1996	1997	1998
Profits	228.1	250.5	316.6	400.0	456.7

In Exercises 9 and 10, plot the points and find the distance between the points.

9. $(-3, 8)$, $(1, 5)$
10. $(5.6, 0)$, $(0, 8.2)$

Geometry **In Exercises 11 and 12, plot the points and verify that the points form the polygon.**

11. *Right Triangle:* $(2, 3)$, $(13, 11)$, $(5, 22)$

12. *Parallelogram:* $(1, 2)$, $(8, 3)$, $(9, 6)$, $(2, 5)$

In Exercises 13 and 14, plot the points and find the midpoint of the line segment joining the points.

13. $(-12, 5)$, $(4, -7)$

14. $(1.8, 7.4)$, $(-0.6, -14.5)$

15. ***Business*** The Sbarro restaurant chain had revenues of \$329.5 million in 1996 and \$375.2 million in 1998. (Source: Sbarro, Inc.)

(a) Without any additional information, what would you estimate the 1997 revenues to have been?

(b) The actual revenue for 1997 was \$349.4 million. How accurate is your estimate?

In Exercises 16 and 17, find the standard form of the equation of the specified circle.

16. Center: $(3, -1)$; Solution point: $(-5, 1)$

17. End points of a diameter: $(-4, 6)$, $(10, -2)$

P.2 In Exercises 18 and 19, complete the table. Use the resulting solution points to sketch the graph of the equation.

18. $y = -\frac{1}{2}x + 2$

x	-2	0	2	3	4
y					

19. $y = x^2 - 3x$

x	-1	0	1	2	3
y					

In Exercises 20–29, sketch the graph of the equation *by hand*.

20. $y - 2x - 3 = 0$
21. $3x + 2y + 6 = 0$
22. $x - 5 = 0$
23. $y = 8 - |x|$
24. $y = \sqrt{5 - x}$
25. $y = \sqrt{x + 2}$
26. $y + 2x^2 = 0$
27. $y = x^2 - 4x$
28. $x + y^2 = 9$
29. $x^2 + y^2 = 10$

In Exercises 30–37, use a graphing utility to graph the equation. Approximate any intercepts.

30. $y = \frac{1}{4}(x + 1)^3$
31. $y = 4 - (x - 4)^2$
32. $y = \frac{1}{4}x^4 - 2x^2$
33. $y = \frac{1}{4}x^3 - 3x$
34. $y = x\sqrt{9 - x^2}$
35. $y = x\sqrt{x + 3}$
36. $y = |x - 4| - 4$
37. $y = |x + 2| + |3 - x|$

38. ***Data Analysis*** The average expenditures y (in dollars) for automobile insurance per insured vehicle from 1992 through 1996 are shown in the table. (Source: National Association of Insurance Commissioners)

x	1992	1993	1994	1995	1996
y	616	638	651	667	685

(a) Use a graphing utility to plot the data.
(b) Use the regression capabilities of a graphing utility to find the best-fitting linear model (let $t = 2$ correspond to 1992).
(c) Graph the model in the same viewing window with the data.
(d) Use the model to estimate the values of y for the years 2000 and 2002.

P.3 In Exercises 39–44, plot the two points and find the slope of the line that passes through the points.

39. $(-3, 2), (8, 2)$
40. $(7, -1), (7, 12)$
41. $\left(\frac{3}{2}, 1\right), \left(5, \frac{5}{2}\right)$
42. $\left(-\frac{3}{4}, \frac{5}{6}\right), \left(\frac{1}{2}, -\frac{5}{2}\right)$
43. $(-4.5, 6), (2.1, 3)$
44. $(-2.7, -6.3), (-1, -1.2)$

In Exercises 45–48, use the concept of slope to find t such that the three points are collinear.

45. $(-2, 5), (0, t), (1, 1)$
46. $(-6, 1), (1, t), (10, 5)$
47. $(1, -4), (t, 3), (5, 10)$
48. $(-3, 3), (t, -1), (8, 6)$

In Exercises 49–58, (a) find an equation of the line that passes through the given point and has the specified slope, and (b) find three additional points through which the line passes.

	Point	*Slope*
49.	$(2, -1)$	$m = \frac{1}{4}$
50.	$(-3, 5)$	$m = -\frac{3}{2}$
51.	$(0, -5)$	$m = \frac{3}{2}$
52.	$(3, 0)$	$m = -\frac{2}{3}$
53.	$\left(\frac{1}{5}, -5\right)$	$m = -1$
54.	$\left(0, \frac{7}{8}\right)$	$m = -4$
55.	$(-2, 6)$	$m = 0$
56.	$(-8, 8)$	$m = 0$
57.	$(10, -6)$	m is undefined.
58.	$(5, 4)$	m is undefined.

In Exercises 59–64, (a) find an equation of the line (in slope-intercept form) that passes through the points and (b) sketch the graph of the equation.

59. $(2, -1), (4, -1)$
60. $(0, 0), (0, 10)$
61. $(2, 1), (14, 6)$
62. $(-2, 2), (3, -10)$
63. $(-1, 0), (6, 2)$
64. $(1, 6), (4, 2)$

In Exercises 65–68, write equations of the lines through the point (a) parallel to the given line and (b) perpendicular to the given line. Verify your result with a graphing utility (use a square setting).

	Point	*Line*
65.	$(3, -2)$	$5x - 4y = 8$
66.	$(-8, 3)$	$2x + 3y = 5$
67.	$(-6, 2)$	$x = 4$
68.	$(3, -4)$	$y = 2$

P.4 In Exercises 69–72, solve the equation (if possible) and use a graphing utility to verify your solution.

69. $14 + \dfrac{2}{x - 1} = 10$
70. $6 - \dfrac{11}{x} = 3 + \dfrac{7}{x}$
71. $\dfrac{9x}{3x - 1} - \dfrac{4}{3x + 1} = 3$
72. $\dfrac{5}{x - 5} + \dfrac{1}{x + 5} = \dfrac{2}{x^2 - 25}$

In Exercises 73–76, determine the x- and y-intercepts of the graph of the equation algebraically. Use a graphing utility to verify your answer.

73. $-x + y = 3$
74. $x - 5y = 20$
75. $y = x^2 - 9x + 8$
76. $y = 25 - x^2$

In Exercises 77 and 78, use a graphing utility to graph the equation and approximate the x- and y-intercepts.

77. $y = -|x + 5| - 2$
78. $y = 6 - 2|x - 3|$

In Exercises 79–84, use a graphing utility to approximate any solutions (accurate to three decimal places) of the equation.

79. $5(x-2)-1=0$
80. $12-5(x-7)=0$
81. $3x^3-2x+4=0$
82. $\frac{1}{3}x^3-x+4=0$
83. $x^4-3x+1=0$
84. $6-\frac{1}{2}x^2+\frac{5}{6}x^4=0$

In Exercises 85–88, determine algebraically any points of intersection of the graphs of the equations. Use a graphing utility to verify your answer(s).

85. $\begin{aligned} 3x+5y&=-7\\ -x-2y&=3\end{aligned}$
86. $\begin{aligned} x-y&=3\\ 2x+y&=12\end{aligned}$
87. $\begin{aligned} x^2+2y&=14\\ 3x+4y&=1\end{aligned}$
88. $\begin{aligned} y&=-x+7\\ y&=2x^3-x+9\end{aligned}$

In Exercises 89–98, use any method to solve the equation. Use a graphing utility to verify your solution(s).

89. $6x=3x^2$
90. $15+x-2x^2=0$
91. $(x+4)^2=18$
92. $16x^2=25$
93. $x^2-12x+30=0$
94. $x^2+6x-3=0$
95. $2x^2+9x-5=0$
96. $-x^2-x+15=0$
97. $x^2-4x-10=0$
98. $-2x^2-13x=0$

In Exercises 99–116, solve the equation (if possible) and use a graphing utility to verify your solution.

99. $3x^3-26x^2+16x=0$
100. $216x^4-x=0$
101. $5x^4-12x^3=0$
102. $4x^3-6x^2=0$
103. $\sqrt{x+4}=3$
104. $\sqrt{x-2}-8=0$
105. $\sqrt{2x+3}+\sqrt{x-2}=2$
106. $5\sqrt{x}-\sqrt{x-1}=6$
107. $(x-1)^{2/3}-25=0$
108. $(x+2)^{3/4}=27$
109. $3\left(1-\frac{1}{5t}\right)=0$
110. $\frac{1}{x-2}=3$
111. $\frac{4}{(x-4)^2}=1$
112. $\frac{1}{(t+1)^2}=1$
113. $|x-5|=10$
114. $|2x+3|=7$
115. $|x^2-3|=2x$
116. $|x^2-6|=x$

P.5 In Exercises 117–134, solve the inequality and graph the solution on the real number line. Use a graphing utility to verify your solution.

117. $8x-3<6x+15$
118. $\frac{1}{2}(3-x)>\frac{1}{3}(2-3x)$
119. $-2<-x+7\le 10$
120. $-6\le 3-2(x-5)<14$
121. $|x-2|<1$
122. $|x|\le 4$
123. $\left|x-\frac{3}{2}\right|\ge\frac{3}{2}$
124. $|x-3|>4$
125. $4|3-2x|\le 16$
126. $|x+9|+7>19$
127. $x^2-2x\ge 3$
128. $4x^2-23x\le 6$
129. $x^3-16x\ge 0$
130. $12x^3-20x^2<0$
131. $\frac{x-5}{3-x}<0$
132. $\frac{2}{x+1}\le\frac{3}{x-1}$
133. $\frac{3x+8}{x-3}\le 4$
134. $\frac{x+8}{x+5}-2<0$

135. *Accuracy of Measurement* The side of a square is measured as 20.8 inches with a possible error of $\frac{1}{16}$ inch. Using these measurements, determine the interval containing the area of the square.

Synthesis

True or False? **In Exercises 136 and 137, determine whether the statement is true or false. Justify your answer.**

136. If $ab=0$, then the point (a, b) lies on the x-axis or on the y-axis.

137. The graph of an equation may have two distinct y-intercepts.

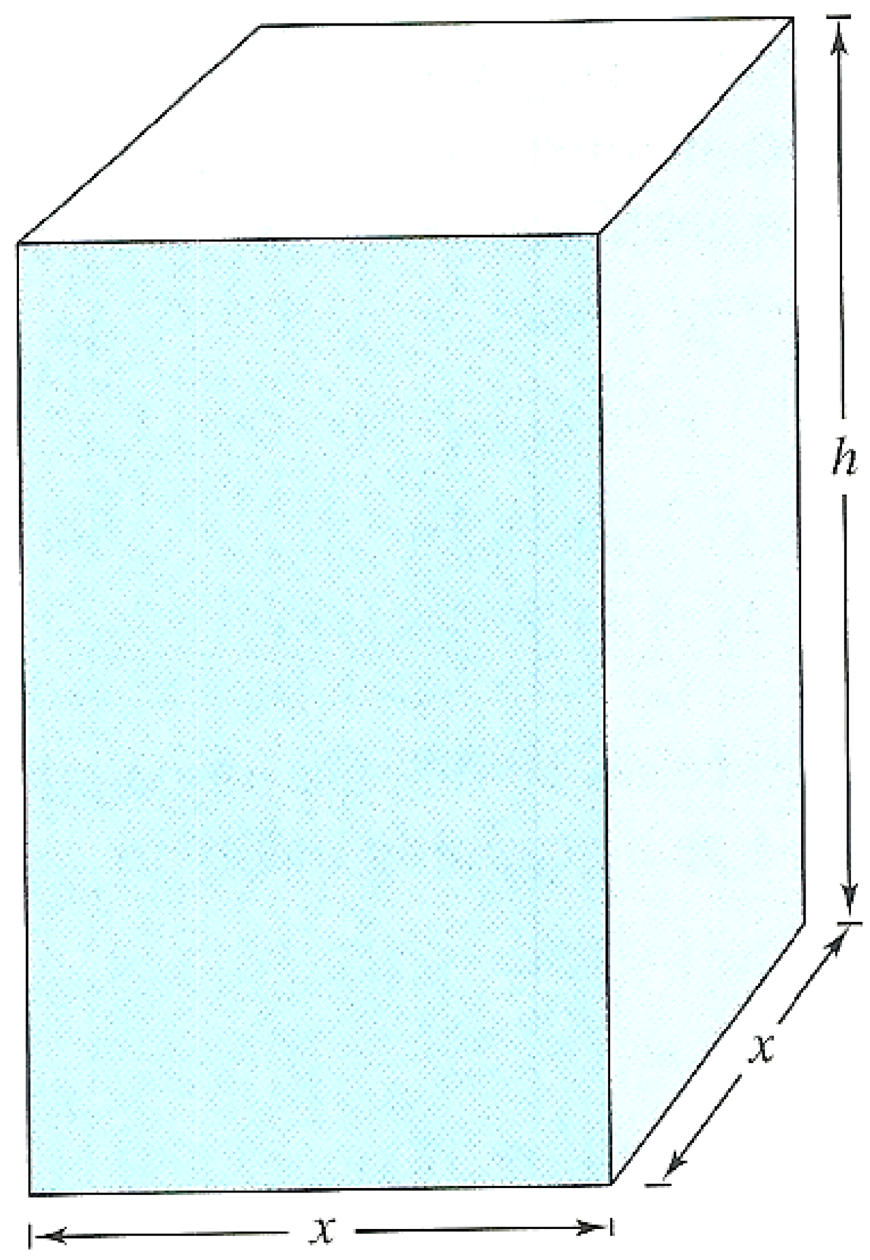
138. In your own words, explain the difference between an identity and a conditional equation.

139. In your own words, explain what is meant by equivalent equations. Describe the steps used to transform an equation into an equivalent equation.

Chapter Project *Modeling the Volume of a Box*

Many mathematical results are discovered experimentally by calculating examples and looking for patterns. Prior to the 1950s, this mode of discovery was very time-consuming because the calculations had to be done by hand. The introduction of computer and calculator technology has removed much of this drudgery. In the following project, you are asked to model a real-life situation and solve a problem by looking for patterns in the corresponding data.

Consider a rectangular box with a square base and a surface area of 216 square inches. Let x represent the length (in inches) of each side of the base and let h represent the height (in inches) of the box, as shown at the right. Your goal is to answer this question: "Of all rectangular boxes with square bases and surface area of 216 square inches, which has the greatest volume?"



A computer simulation to accompany this project appears in the *Interactive* CD-ROM and *Internet* versions of this text.

a. Express the areas of the base, top, and sides in terms of x and h.

b. Find an expression in terms of x and h for the surface area of the box.

c. Use the fact that the surface area is 216 square inches to express the variable h in terms of x.

d. Find an expression for the volume of the box in terms of x alone.

e. Use the expression in part (d) and a graphing utility to complete the table. Then use the results to decide which box has the greatest volume.

Base, x	*Height, h*	*Surface Area*	*Volume*
1.0	53.5	216.0	53.5
1.5	35.3	216.0	79.3
2.0	26.0	216.0	104.0
⋮	⋮	⋮	⋮
10.0	0.4	216.0	40.0

Questions for Further Exploration

1. What happens to the height of the box as x gets closer and closer to 0? Of all boxes with square bases and a surface area of 216 square inches, is there a tallest? Explain your reasoning.
2. What is the maximum value of x? What happens to the height of the box as x gets closer and closer to this maximum value? Is there a shortest box that has a square base and a surface area of 216 square inches? Explain your reasoning.
3. Complete the table. Does it lend further support to your answer to part (e)? Explain.

x	5.9	5.99	5.999	6.001	6.01	6.1
V						

4. Of all rectangular boxes with surface area of 216 square inches and a base that is x inches by $2x$ inches, which has the maximum volume? Explain your reasoning.

P Chapter Test

Take this test as you would take a test in class. After you are done, check your work against the answers in the back of the book.

The *Interactive* CD-ROM and *Internet* versions of this text provide answers to the Chapter Tests and Cumulative Tests. They also offer Chapter Pre-Tests (which test key skills and concepts covered in previous chapters) and Chapter Post-Tests, both of which have randomly generated exercises with diagnostic capabilities.

1. Plot the points $(-2, 5)$ and $(6, 0)$. Find the coordinates of the midpoint of the line segment joining the points and the distance between the points.
2. The numbers (in millions) of votes cast for the Democratic candidates for president in 1980, 1984, 1988, 1992, and 1996 were 35.5, 37.6, 41.8, 44.9, and 47.4, respectively. Create a bar graph for this data. (Source: Congressional Quarterly, Inc.)
3. Find the standard form of the equation of a circle with center $(4, -1)$ and a solution point $(1, 4)$.

In Exercises 4–9, use the point-plotting method to graph the equation and identify any intercepts. Verify your results using a graphing utility.

4. $y = 4 - \frac{3}{4}|x|$
5. $y = 4 - (x - 2)^2$
6. $y = x - x^3$
7. $y = -x^3 + 2x - 4$
8. $y = \sqrt{3 - x}$
9. $y = \frac{1}{2}x\sqrt{x + 3}$
10. A line passes through the point $(3, -1)$ with slope $m = \frac{3}{2}$. Write the equation of the line. List three additional points on the line. Then sketch the line.
11. Find an equation of the line that passes through the point $(0, 4)$ and is perpendicular to the line $5x + 2y = 3$.
12. Solve $\dfrac{12}{x} - 7 = -\dfrac{27}{x} + 6$ and use a graphing utility to verify your solution.

In Exercises 13–15, use a graphing utility to graph the equation. Determine the number of x-intercepts of the graph.

13. $y = 3x^2 + 1$
14. $y = x^3 + x$
15. $y = x^3 - 4x^2 + 5x$

In Exercises 16–21, find all solutions of the equation. Check your solution(s) algebraically and graphically.

16. $x^2 - 10x + 9 = 0$
17. $4x^2 - 81 = 0$
18. $3x^3 - 4x^2 - 12x + 16 = 0$
19. $x + \sqrt{22 - 3x} = 6$
20. $(x^2 + 6)^{2/3} = 16$
21. $|8x - 1| = 21$

In Exercises 22–24, solve the inequality and sketch the solution on the real number line. Use a graphing utility to verify your solution.

22. $-\frac{5}{6} < x - 2 < \frac{1}{8}$
23. $2|x - 8| < 10$
24. $\dfrac{3 - 5x}{2 + 3x} < -2$
25. The table at the right shows the number of local telephone calls C (in billions) in the United States from 1989 through 1996, where $t = 0$ represents 1990. Use the regression capabilities of a graphing utility to find a linear model for the data. Determine algebraically and graphically when the annual number of local calls will be about 600 billion. (Source: U.S. Federal Communications Commission)

t	C
-1	389
0	402
1	416
2	434
3	447
4	465
5	484
6	504

LIBRARY OF FUNCTIONS

In Chapter 1, you will be introduced to the concept of a *function.* As you proceed through the text, you will see that functions play a primary role in modeling real-life situations.

Over the past few hundred years, many different types of functions have been introduced and studied. Those that have proven to be most important in modeling real life have come to be known as *elementary functions.* There are three basic types of elementary functions: algebraic functions, exponential and logarithmic functions, and trigonometric and inverse trigonometric functions.

You will also encounter other types of functions in this text, such as functions defined by real-life data and piecewise-defined functions.

Library of Functions

Each time a new type of function is studied in detail in this text, it will be highlighted in a box like this one. For instance, a linear function is highlighted in the Library of Functions box in Section P.3 because lines are discussed in that section.

In addition, there is a Library of Functions Summary inside the front cover which describes the functions listed below.

Algebraic Functions	
Polynomial Functions	
Linear or First-Degree Polynomial Functions	Section P.3
Quadratic or Second-Degree Polynomial Functions	Section 2.1
Cubic or Third-Degree Polynomial Functions	Section 2.2
Fourth- and Higher-Degree Polynomial Functions	Section 2.2
Rational Functions	Section 2.6
Radical Functions	Section 1.1
Exponential and Logarithmic Functions	
Exponential Functions	Section 3.1
Logarithmic Functions	Section 3.2
Trigonometric and Inverse Trigonometric Functions	
Trigonometric Functions	Section 4.2
Inverse Trigonometric Functions	Section 4.7
Nonelementary Functions	
Piecewise-Defined Functions	Section 1.1
Greatest Integer Functions	Section 1.2
Absolute Value Functions	Section 1.2
Functions Defined by Real-Life Data	Section 1.1