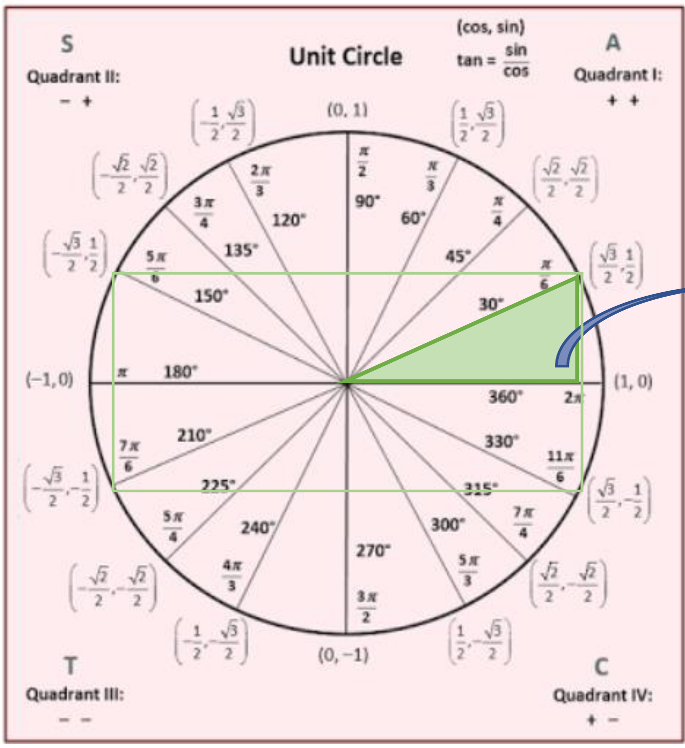
NAME: ______________________________ PLEASE WRITE YOUR NAME ON <u>ALL</u> PAGES.
TEACHER: I MADARANG
SUBJECT: ALGEBRA 2 WEEK 4 Due: May 15th

PERIOD: __________________

WEEK 3: THE TRIGONOMETRIC IDENTITIES

<u>PART I: The Trigonometric Table</u>. Last week you were asked to fill in the unit circle. What does this have to do with the trigonometric identities, sine, cosine and tangent?

Let's look at the unit circle again:

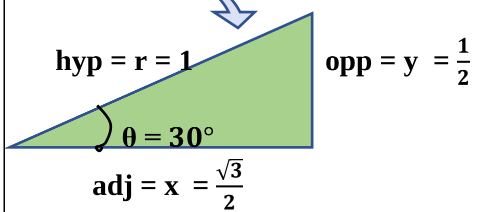


Is this how you filled the unit circle last week? Take out your unit circle and check your answers.😊 Be mindful of the +/- values of your coordinates.

Now let's connect this to your sine, cosine and tangent identities. NOTE: r = 1 because this is a UNIT circle, hence 1 unit

SOCAHTOA: **Sine:** $S = \frac{Opp}{Hyp}$ **Cosine:** $C = \frac{Adj}{Hyp}$

Tangent: $T = \frac{Opp}{Adj}$



As you can see, with respect to the central ∠30°,

$sin\ 30° = \frac{y}{r} = \frac{\frac{1}{2}}{1} = \boxed{\frac{1}{2}}$ → note that this is just the y-value of 30°

$cos\ 30° = \frac{x}{r} = \frac{\frac{\sqrt{3}}{2}}{1} = \boxed{\frac{\sqrt{3}}{2}}$ → this is just the x-value of 30°

$\tan 30° = \frac{y}{x} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ → substituting x and y

$= \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$ → copy, dot, flip

$= \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ → rationalize

$= \boxed{\frac{\sqrt{3}}{3}}$ → simplify

Do you need to solve for every angle related to ∠30° ? OF COURSE . . .NOT! Surely you remember the rectangles? All you need to do is change the signs according to the location on the unit circle. Ok, so fill this table.

Degrees	Radians	Sin = $\frac{y}{r}$	Cos = $\frac{x}{r}$	Tan = $\frac{y}{x}$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
150°				$-\frac{\sqrt{3}}{3}$
		$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	
	$\frac{11\pi}{6}$			

NAME ____ PERIOD ___________________

So basically, what work do you need to show here? You just have to show me how you will find the tangent value of the 60°, 45° and the axis angles and you should be able to get ALL trig ratios of your unit circle!

$$\tan 60° = \frac{y}{x} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$$

What are the values for:

$\tan 120° = \boxed{\quad}$ $\tan 240° = \boxed{\quad}$ $\tan 300° = \boxed{\quad}$

$$\tan 45° = \frac{y}{x} = \frac{\frac{\sqrt{2}}{2}}{\boxed{\quad}} = \boxed{\quad}$$

What are the values for:

$\tan 135° = \boxed{\quad}$ $\tan 225° = \boxed{\quad}$ $\tan 315° = \boxed{\quad}$

$$\tan 0° = \tan 360° = \frac{y}{x} = \frac{o}{1} = \boxed{\quad}$$

(What's $\frac{0}{1}$?)

$$\tan 90° = \frac{y}{x} = \frac{1}{0} = \boxed{\quad}$$

(What's $\frac{1}{0}$? It's a word, not a number!)

$$\tan 180° = \frac{y}{x} = \boxed{\quad}$$

$$\tan 270° = \frac{y}{x} = \boxed{\quad}$$

GREAT!! Now let's put all those values in the trig table below. Go around the unit circle as you fill in the values.

Degrees	Radians	Sin = $\frac{y}{r}$	Cos = $\frac{x}{r}$	Tan = $\frac{y}{x}$
0° or 360°	0π or 2π	0	1	0
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$
45°				
60°				
90°		1	0	undefined
	$\frac{2\pi}{3}$			
	$\frac{3\pi}{4}$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1
	$\frac{5\pi}{6}$			
	π			
210°				
225°				
240°		$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	1
270°				
	$\frac{5\pi}{3}$			
	$\frac{7\pi}{4}$			
	$\frac{11\pi}{6}$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$

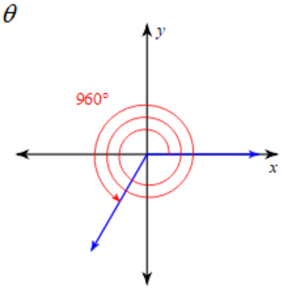
NAME ____ PERIOD ______________________

PART IIA: EVALUATING THE TRIG FUNCTION BASED ON THE COTERMINAL ANGLE (in degrees).
Identify the coterminal angle, then determine the trig function. NOTE: Coterminus means it ends in the same point.

Find the exact value of each trigonometric function.

EXAMPLE 1:

$\tan\theta$



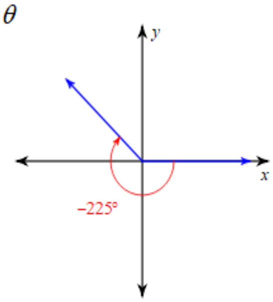
Since $\theta = 960°$ is greater than 360°, we need to find its reference angle. First we have to find how many revolutions do we have to make to reach $960°$. From the diagram, you need 2 whole revolution and a partial revolution. To find the exact value we subtract 2(360) from 960.

$\boxed{960 - 2(360) = 240°}$

Our coterminal angle is 240°. Next from our table on the second page, we find $\tan 240°$. The table shows that $\tan 240° = \sqrt{3}$. So there you go, our answer is : $\boxed{\tan 240° = \sqrt{3}}$

EXAMPLE 2: You can do the same for negative angles. Just find the coterminal angle, then the trig function.

$\sin\theta$

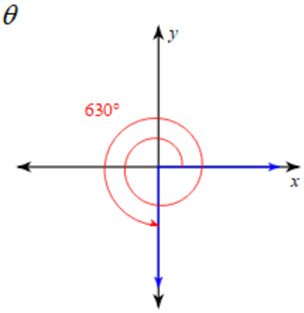


ANSWER: Since $-225 + 1(360) = 135°$, and from the table $\boxed{\sin 135° = \frac{\sqrt{2}}{2}}$ → **ANSWER!**

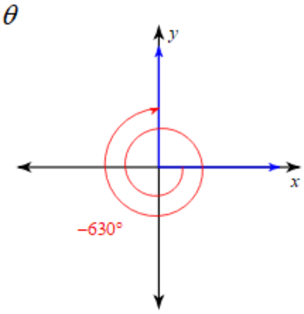
NOTE: If the diagram is not given to you, you have to figure out how many revolutions (or counter revolutions) it would take for you to get the coterminal angle that is between 0 and 360.

Find the exact value of each trigonometric function.

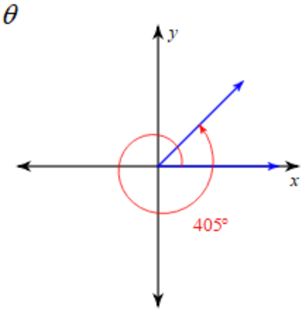
1) $\tan\theta$



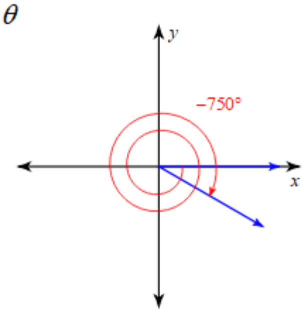
2) $\cos\theta$



3) $\sin\theta$



4) $\cos\theta$



5) $\cos -240°$

6) $\tan 630°$

7) $\cos -855°$

8) $\tan 420°$

9) $\sin 660°$

10) $\tan 855°$

NAME ____ PERIOD ________________

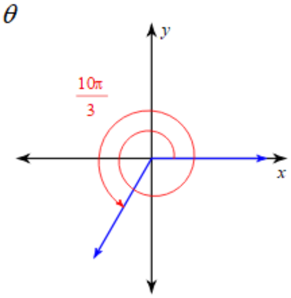
PART IIB: EVALUATING THE TRIG FUNCTION BASED ON THE COTERMINAL ANGLE (in radians).

Identify the coterminal angle, then determine the trig function.

Find the exact value of each trigonometric function.

EXAMPLE 3:

$\sin\,\theta$



You will do the same thing here as in Ex 1, except that you will have to either add or subtract $2n\pi$ where n is the number of revolutions.

So, $\frac{10\pi}{3} - 2\pi = \frac{10\pi}{3} - \frac{6\pi}{3}$
$= \frac{4\pi}{3}$ → coterminal angle

Therefore,

$$\boxed{\sin\frac{4\pi}{3} = -\frac{\sqrt{3}}{2}}$$

EXAMPLE 2:

Find $\cos\left(-\frac{10\pi}{3}\right)$

Work:

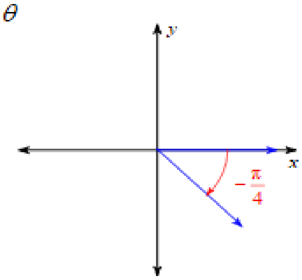
$-\frac{10\pi}{3} + 2(2\pi) = -\frac{10\pi}{3} + \frac{12\pi}{3} = \frac{2\pi}{3}$ → coterminal angle

Therefore:

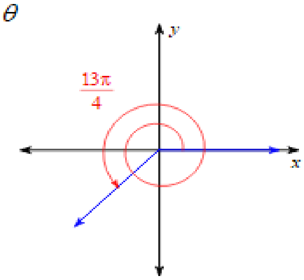
$$\boxed{\cos\frac{2\pi}{3} = -\frac{1}{2}}$$

Find the value of each trig function:

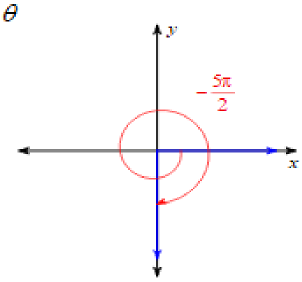
11) $\cos\,\theta$



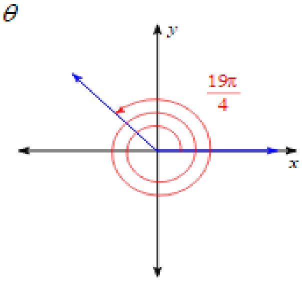
12) $\tan\,\theta$



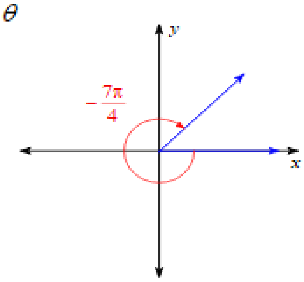
13) $\sin\,\theta$



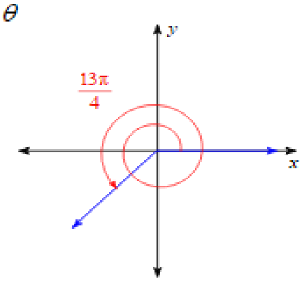
14) $\sin\,\theta$



15) $\tan\,\theta$



16) $\sin\,\theta$



17) $\cos -\frac{21\pi}{4}$

18) $\tan -\frac{9\pi}{4}$

19) $\cos \frac{3\pi}{2}$

20) $\cos -\frac{2\pi}{3}$