NAME:

TEACHER: I MADARANG

SUBJECT: ADVANCED MATH

PERIOD 3 WEEK 1

LESSON 1: INVERSE TRIGONOMETRIC FUNCTIONS

Definitions of the Inverse Trigonometric Functions

Function	*Domain*	*Range*
$y = \arcsin x$ if and only if $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \arccos x$ if and only if $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \arctan x$ if and only if $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

What does this mean? This means that your given ratios will only be limited to the domain and your answers should only be limited to the range. For example, if you are asked to find arccos (-1.5) or $\cos^{-1}$(1.5), your answer will be undefined because the -1.5 does not belong to the domain for cosine.

EXAMPLE 1 Evaluating the Inverse Sine Function

If possible, find the exact value.

a. $\arcsin\left(-\frac{1}{2}\right)$ **b.** $\sin^{-1}\frac{\sqrt{3}}{2}$ **c.** $\sin^{-1} 2$

Solution

a. Because $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$, and $-\frac{\pi}{6}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, it follows that

$$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}. \qquad \text{Angle whose sine is } -\tfrac{1}{2}$$

b. Because $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, and $\frac{\pi}{3}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, it follows that

$$\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}. \qquad \text{Angle whose sine is } \sqrt{3}/2$$

c. It is not possible to evaluate $y = \sin^{-1} x$ at $x = 2$ because there is no angle whose sine is 2. Remember that the domain of the inverse sine function is $[-1, 1]$.

EXAMPLE 3 Evaluating Inverse Trigonometric Functions

Find the exact value.

a. $\arccos\frac{\sqrt{2}}{2}$

b. $\cos^{-1}(-1)$

c. $\arctan 0$

d. $\tan^{-1}(-1)$

Solution

a. Because $\cos(\pi/4) = \sqrt{2}/2$, and $\pi/4$ lies in $[0, \pi]$, it follows that

$$\arccos\frac{\sqrt{2}}{2} = \frac{\pi}{4}. \qquad \text{Angle whose cosine is } \tfrac{\sqrt{2}}{2}$$

b. Because $\cos \pi = -1$, and π lies in $[0, \pi]$, it follows that

$$\cos^{-1}(-1) = \pi. \qquad \text{Angle whose cosine is } -1$$

c. Because $\tan 0 = 0$, and 0 lies in $(-\pi/2, \pi/2)$, it follows that

$$\arctan 0 = 0. \qquad \text{Angle whose tangent is } 0$$

d. Because $\tan(-\pi/4) = -1$, and $-\pi/4$ lies in $(-\pi/2, \pi/2)$, it follows that

$$\tan^{-1}(-1) = -\frac{\pi}{4}. \qquad \text{Angle whose tangent is } -1$$

What does this mean? This means that this time you will be given the value and your task is to find which angle it belongs to.

NOTE: $arcsin\ \theta$ and $\sin^{-1}\theta$ mean the same. They will be used interchangeably throughout this unit.

Find the exact value of each expression. You may use your unit circle or trig table of values. Write your answer in radians. You may use your trig table or unit circle to get the answer.

1) $\cos^{-1} -1$

2) $\csc^{-1} 1$

3) $\tan^{-1} -\frac{\sqrt{3}}{3}$

4) $\cot^{-1} \frac{\sqrt{3}}{3}$

5) $\tan^{-1} 1$

6) $\sec^{-1} -1$

7) $\sin^{-1} -\frac{\sqrt{3}}{2}$

8) $\cot^{-1} -\frac{\sqrt{3}}{3}$

9) $\cot^{-1} \sqrt{3}$

10) $\sin^{-1} \frac{1}{2}$

LESSON 2: USING INVERSE PROPERTIES IN COMPOSITION OF TRIGONOMETRIC FUNCTIONS

EXAMPLE 6 Using Inverse Properties

If possible, find the exact value.

a. $\tan[\arctan(-5)]$ **b.** $\arcsin\left(\sin \frac{5\pi}{3}\right)$ **c.** $\cos(\cos^{-1} \pi)$

Solution

a. Because -5 lies in the domain of the arctangent function, the inverse property applies, and you have

$$\tan[\arctan(-5)] = -5.$$

b. In this case, $5\pi/3$ does not lie within the range of the arcsine function, $-\pi/2 \le y \le \pi/2$. However, $5\pi/3$ is coterminal with

$$\frac{5\pi}{3} - 2\pi = -\frac{\pi}{3}$$

which does lie in the range of the arcsine function, and you have

$$\arcsin\left(\sin \frac{5\pi}{3}\right) = \arcsin\left[\sin\left(-\frac{\pi}{3}\right)\right] = -\frac{\pi}{3}.$$

c. The expression $\cos(\cos^{-1} \pi)$ is not defined because $\cos^{-1} \pi$ is not defined. Remember that the domain of the inverse cosine function is $[-1, 1]$.

Find the exact value of each expression.

11) $\tan^{-1}\left(\sin -\dfrac{\pi}{2}\right)$

12) $\cos^{-1}\left(\csc \dfrac{\pi}{2}\right)$

13) $\sin^{-1}(\cos 0)$

14) $\cos^{-1}(\tan 0)$

15) $\tan^{-1}(\sec 0)$

16) $\cos^{-1}\left(\tan -\dfrac{\pi}{4}\right)$

17) $\cos^{-1}\left(\cot \dfrac{3\pi}{4}\right)$

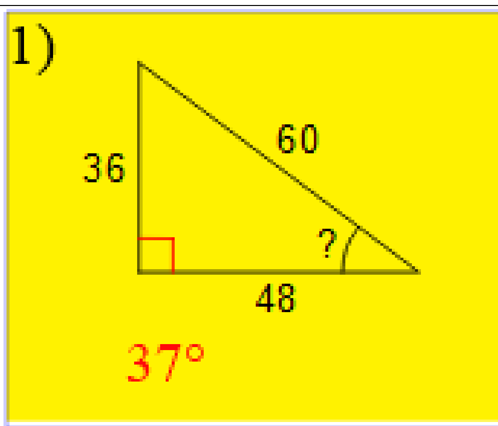
18) $\tan^{-1}(\sec \pi)$

19) $\sin^{-1}\left(\cos \dfrac{\pi}{6}\right)$

20) $\cos^{-1}(\sec \pi)$

To answer #1, first find sin $\left(-\frac{\pi}{2}\right)$, which is -1. Then evaluate $\tan^{-1} -1$, which is $-\frac{\pi}{4}$. so the answer is $-\frac{\pi}{4}$.

LESSON 3: FINDING ANGLE MEASURES IN A RIGHT TRIANGLE USING INVERSE TRIG FUNCTIONS.



Discussion: To find the measure of the indicated angle, first you have to pick two sides and determine the ratio of the sides with respect to the angle. For example, if you pick the sides that measure 36 (opposite) and 60 (hypotenuse), you will be using this equation:

$$\sin \theta = \frac{opposite}{hypotenues} = \frac{36}{60}$$

To find θ, use the inverse trig equation

$$\theta = \sin^{-1}\left(\frac{36}{60}\right)$$

Then use the second function feature of your calculator to find arcsin or $\sin^{-1}$
Your answer should be: $\theta = 36.86°$ **OR 37°.**

(In the exercise below, you may round off your answer to 2 decimal places or round it off to the nearest whole.)

PRACTICE: Using the 2nd function feature of your calculator, find the measure of the indicated angle to the nearest degree. Use inverse trig functions if necessary.

21) $\tan W = 0.4040$

22) $\cos Y = 0.1219$

23) $\sin C = 0.1736$

24) $\sin U = 0.4540$

25) $\cos W = 0.6428$

26) $\tan W = 1.1106$

27) $\sin B = 0.1564$

28) $\cos Y = 0.9563$

29) $\cos X = 0.3256$

30) $\cos Y = 0.7431$

Find the measure of the indicated angle to the nearest degree.

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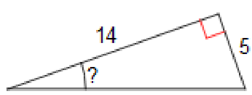

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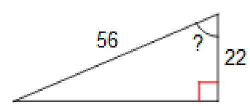

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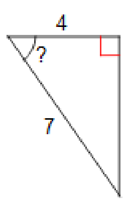

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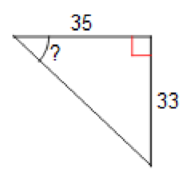

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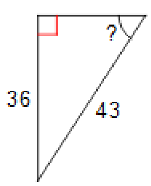

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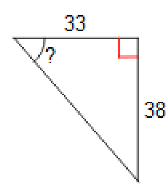

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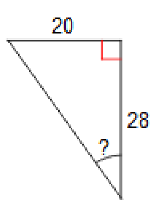

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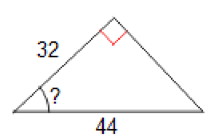

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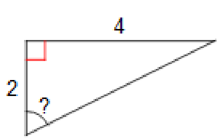

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